Math 10A Final Exam
is **Saturday, Dec 8**
in **here** at 8-11am

Log into TritonEd to view your assigned seat.

Final Exam covers everything we've done

You don't need blue books. Calculators are not allowed. You are allowed one double sided 8.5 by 11 inch page of handwritten notes. Bring your student ID.

Final Exam New Topics

4.2 and 4.6 Absolute max/min (and optimization)

(a) critical numbers (b) word problems (c) absolute max/min

4.3 Local max/min (and shapes of curves)

(a) increasing, decreasing, concave up, concave down (b) inflection, local max/min (c) First and Second derivative test

3.5-3.7 Derivative techniques

(a) implicit differentiation (b) inverse trig derivatives (c) logarithm differentiation

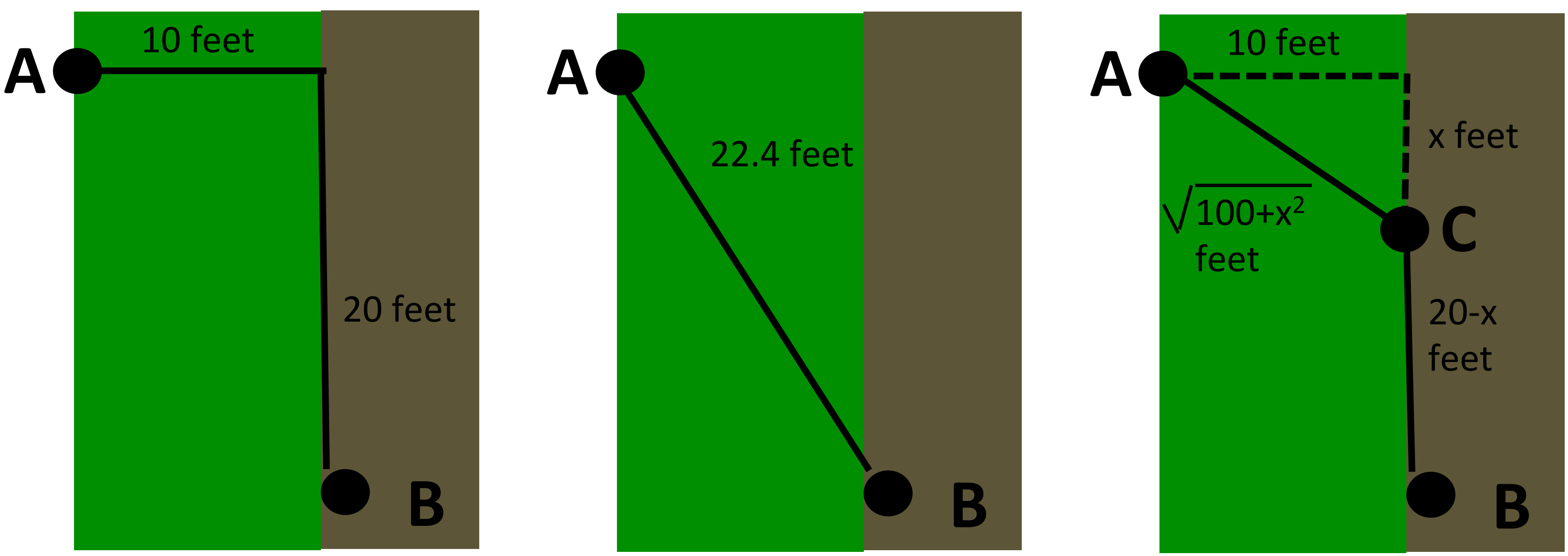
4.5 Limit techniques Limits

(a) indeterminate quotients, products, powers

(b) L'Hopital's Rule

Sec 4.6: Examples of optimization problems

<u>Example</u>: Everyone's favorite pet, Chris Jr. wishes to walk from point A to point B in the picture below. Chris Jr. travels 10 feet/sec over the green grass and 20 feet/sec over the brown dirt trail. What is the minimum amount of time needed to walk from point A to point B?



Sec 4.6: Solutions of optimization problems

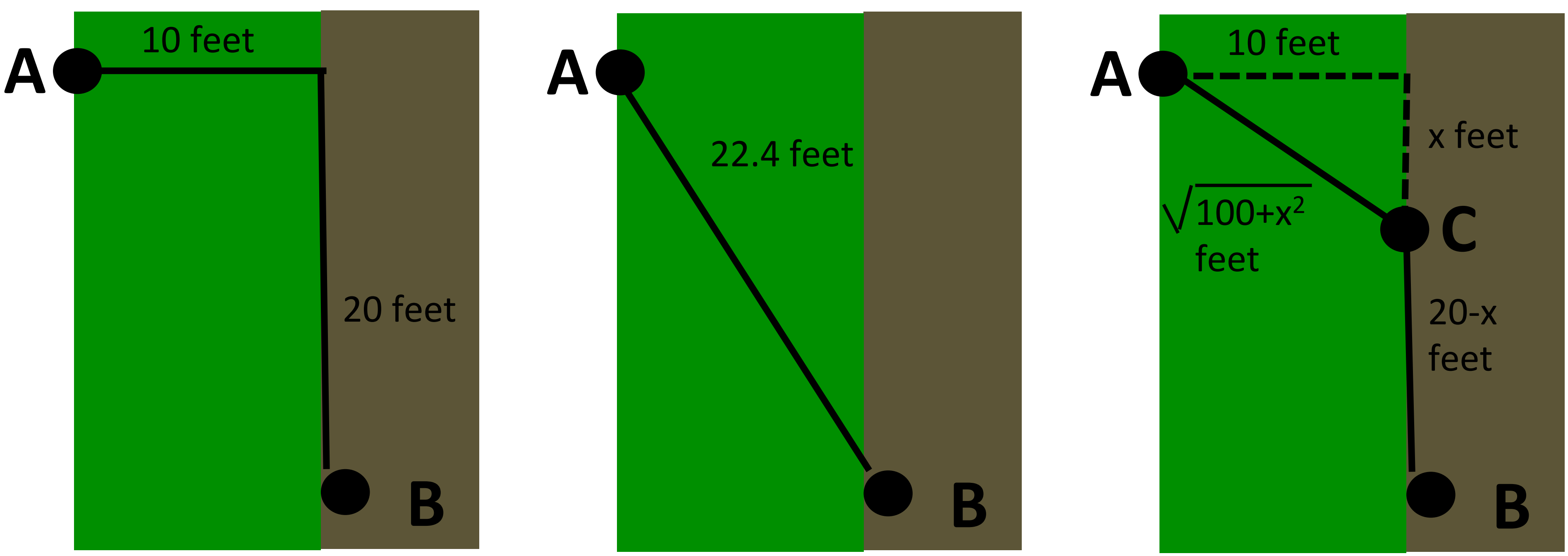
Example: Everyone's favorite pet, Chris Jr. wishes to walk from point A to point B in the picture below. Chris Jr. travels 10 feet/sec over the green grass and 20 feet/sec over the brown dirt trail. What is the minimum amount of time needed to walk from point A to point B?

Then make table!!

type	x	f(x)
endpoint	0	2.00
critical	5.77	**1.87** MIN
endpoint	20	2.24

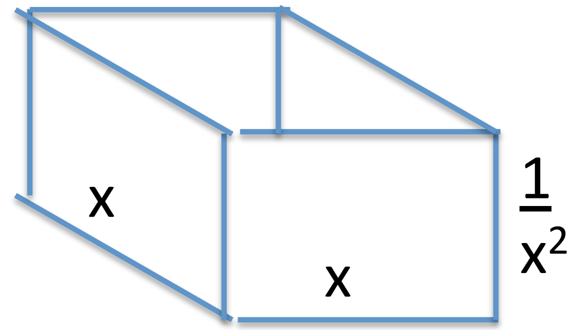
Solution: Convert word problem to equation absolute max/min problem. i.e., Find the absolute minimum of

$$f(x) = \frac{1}{10}\sqrt{100 + x^2} + \frac{1}{20}(20 - x) \text{ on } [0, 20]$$

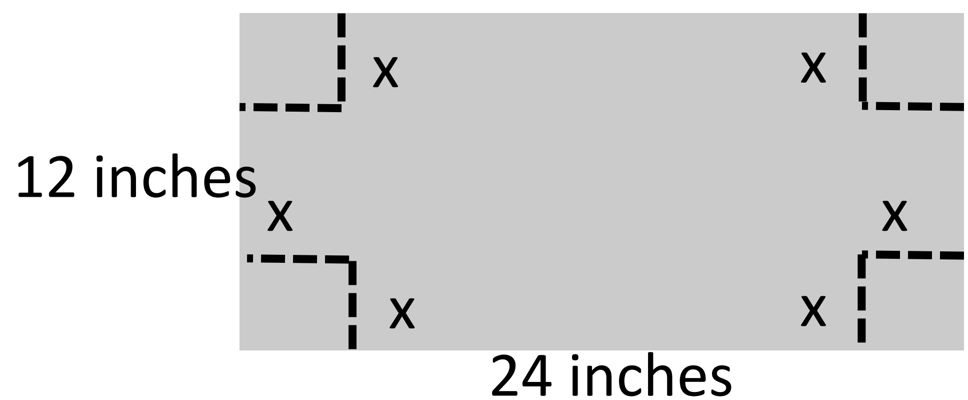


Sec 4.6: Examples of optimization problems

Example 2: Everyone's favorite pet, Chris Jr, would like a rectangular water bowl with volume 1 feet^3 (and no lid). And he wants it to have a square base. What is the minimum surface area of the box (so it's the cheapest for us to build).



Example 3: Everyone's favorite pet, Chris Jr, would like a litter box for our math 10A classroom. We have a 12 inch by 24 inch piece of cardboard. We will cut out the corners as shown below and fold it into a box. What is the maximum volume of the box?



Sec 4.6: Solutions of optimization problems

Example 2: Everyone's favorite pet, Chris Jr, would like a rectangular water bowl with volume 1 feet^3 (and no lid). And he wants it to have a square base. What is the minimum surface area of the box (so it's the cheapest for us to build).

Then table!!

Solution: Convert to equation

Find the absolute minimum of

$$g(x) = x^2 + \frac{4}{x} \text{ on } (0, \infty)$$

type	x	g(x)	
"endpoint"	0	$+\infty$	
critical	1.26	4.76	**MIN**
"endpoint"	∞	$+\infty$	

Example 3: Everyone's favorite pet, Chris Jr, would like a litter box for our math 10A classroom. We have a 12 inch by 24 inch piece of cardboard. We will cut out the corners as shown below and fold it into a box. What is the maximum volume of the box?

Then make a table of critical, endpoint, and f(x) values!!

Solution: Convert to equation

Find the absolute maximum of

$$h(x) = x(24 - 2x)(12 - 2x)$$

$$\text{on } [0, 6]$$

type	x	h(x)	
endpoint	0	0	
critical	2.54	332.5	**MAX**
endpoint	6	0	

Sec 4.2 and 4.6: Examples of absolute max/min problems

Note: You do NOT need 1st and 2nd derivative tests for global max/min

Solution: Make a table of critical, endpoints, and f(x) values!!

1. Find the absolute maximum and absolute minimum of $f(x) = 5 + 54x - 2x^3$ on $[0, 4]$, or show that they do not exist.

2. Find the absolute maximum and absolute minimum of $g(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, \infty)$ or show that they do not exist.

3. Find the absolute maximum and absolute minimum of $h(x) = x^4 - 2x^2 + 3$ on $(-\infty, \infty)$ or show that they do not exist.

4. Find the absolute maximum and absolute minimum of $F(x) = (x^2 - 1)^3$ on $[-1, 2]$, or show that they do not exist.

Sec 4.2 and 4.6: Solutions of absolute max/min problems

1. Find the absolute maximum and absolute minimum of $f(x) = 5 + 54x - 2x^3$ on $[0, 4]$ or show that they do not exist.

type	x	f(x)	
endpoint	0	5	MIN
critical	3	113	MAX
endpoint	4	93	

2. Find the absolute maximum and absolute minimum of $g(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, \infty)$ or show that they do not exist.

type	x	g(x)	
endpoint	MIN	-14	
critical	1	6	
critical	3	2	
"endpoint"	NO MAX	∞	

3. Find the absolute maximum and absolute minimum of $h(x) = x^4 - 2x^2 + 3$ on $(-\infty, \infty)$ or show that they do not exist.

type	x	h(x)	
"endpoint"	-∞	∞	
critical	-1	2	MIN
critical	0	3	
critical	1	2	
"endpoint"	∞	∞	NO MAX

4. Find the absolute maximum and absolute minimum of $F(x) = (x^2 - 1)^3$ on $[-1, 2]$ or show that they do not exist.

type	x	F(x)	
endpoint	-1	0	
critical	0	-1	MIN
critical	1	0	
endpoint	2	27	MAX

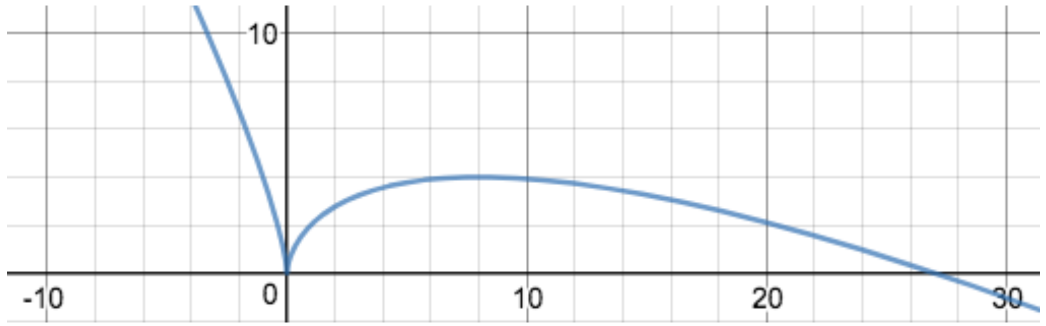
Sec 4.3: Examples of shapes of curves problems

Note: You DO need 1st and 2nd derivative tests for local max/min

1. Suppose $f(x) = 2 + 2x^2 - x^4$, find the intervals where f(x) is increasing, decreasing, concave up, and concave down. Find all the inflection points, local minimum, and local maximum.

2. Suppose $g(x) = 200 + 8x^3 + x^4$, find the intervals where g(x) is increasing, decreasing, concave up, and concave down. Find all the inflection points, local minimum, and local maximum.

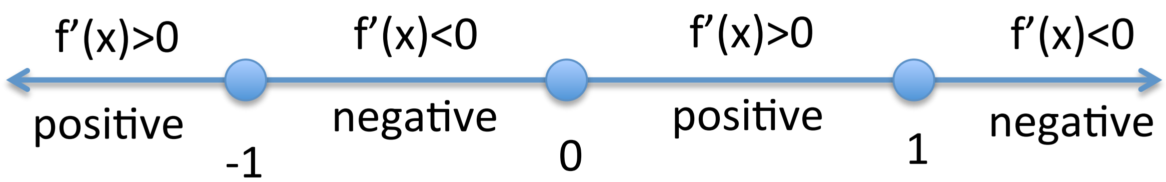
3. Suppose $h(x) = 3x^{\frac{2}{3}} - x$, find the intervals where h(x) is increasing, decreasing, concave up, and concave down. Find all the inflection points, local minimum, and local maximum.



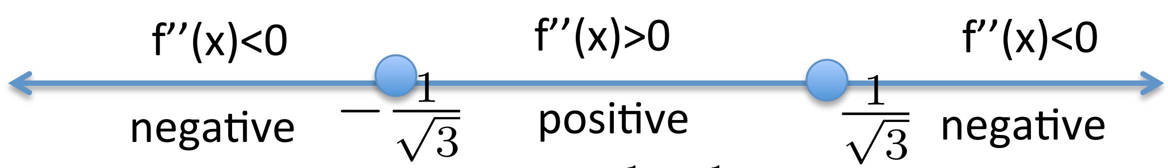
Sec 4.3: Solutions of shapes of curves problems

STEPS for analyzing $f(x) = 2 + 2x^2 - x^4$

1. Compute 1st derivative $f'(x) = 4x - 4x^3 = 4x(x-1)(x+1)$
2. Solve f'(x)=0 for critical numbers of x= -1,0,1
3. Determine if f'(x) is positive or negative in between critical numbers

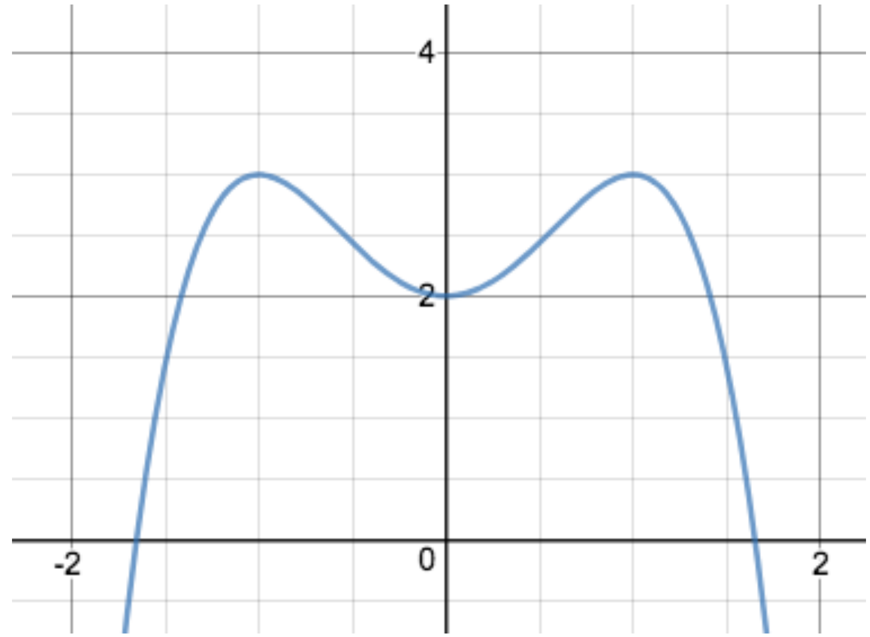


4. Conclude f(x) is **increasing** on (-∞,-1) and (0,1)
   f(x) is **decreasing** on (-1,0) and (0,∞)
5. Classify critical points with first derivative test
   f(x) is **local max** at x= -1, **local min** at x=0, and **local max** at x=1
6. Compute 2nd derivative $f''(x) = 4 - 12x^2 = -12(x - \frac{1}{\sqrt{3}})(x + \frac{1}{\sqrt{3}})$
7. Determine if f''(x) is positive or negative in between f''(x)=0



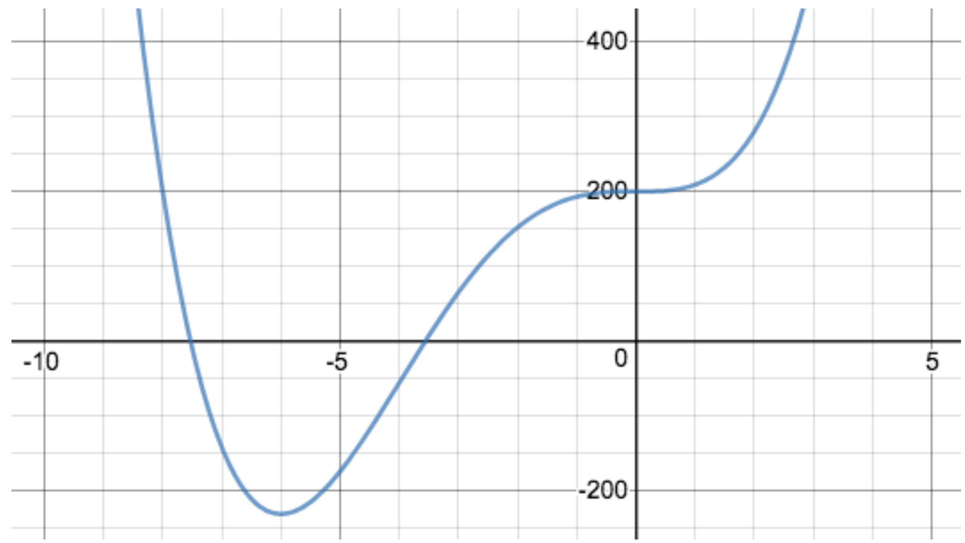
8. Conclude f(x) is **concave up** on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and **down** on $(-\infty, -\frac{1}{\sqrt{3}})$ and $(\frac{1}{\sqrt{3}}, \infty)$
9. Conclude f(x) has **inflection points** at x= -1/$\sqrt{3}$, and 1/$\sqrt{3}$

Sec 4.3: Solutions of shapes of curves problems

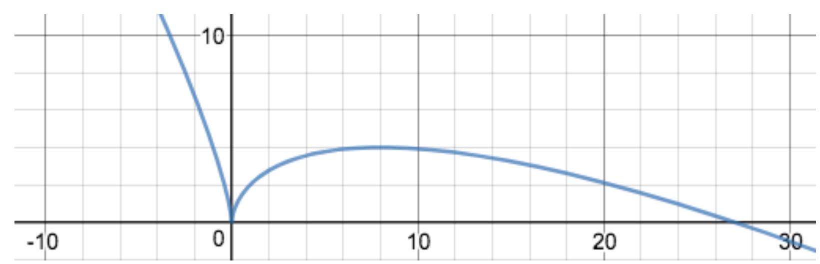


1. On (-∞,1) f(x) is increasing
   on (1,0) f(x) is decreasing
   on (0,1) f(x) is increasing
   on (1,∞) f(x) is decreasing
   By first derivative test,
   f(x) has local max at x=-1
   f(x) has local min at x=0
   f(x) has local max at x=1
   On (-∞,-$\frac{1}{\sqrt{3}}$) f(x) is concave down
   on (-$\frac{1}{\sqrt{3}}$,$\frac{1}{\sqrt{3}}$) f(x) is concave up
   on ($\frac{1}{\sqrt{3}}$, ∞) f(x) is concave down
   f(x) has inflection points at x= -$\frac{1}{\sqrt{3}}$, and $\frac{1}{\sqrt{3}}$

Sec 4.3: Solutions of shapes of curves problems



2. On (-∞,-6) g(x) is decreasing
on (-6, ∞) g(x) is increasing
By first derivative test,
g(x) has local min at x=-6
g(x) has no local max
On (-∞,-4) g(x) is concave up
on (-4,0) g(x) is concave down
on (0,∞) g(x) is concave up
g(x) has inflection points at x= -4 and 0



3. On (-∞,0) h(x) is decreasing
on (0,8) h(x) is increasing
on (8,∞) h(x) is decreasing
By 1st derivative test, h(x) has local min at x=0 and local max at x=8
On (-∞,0) and (0,∞) h(x) is concave down
h(x) has no inflection points

Sec 4.2,4.3,4.6: Examples of global/local max/min

Q1: For each of the functions here, find the absolute (global) maximum and absolute (global) minimum from the table of values.

Q2: Can we find the local maximums and local minimums without the first and second derivative test?

f(x) on [1,7]

type	x	f(x)
endpoint	1	8
critical	3	-1
critical	6	6
endpoint	7	5

g(x) on [-3,∞)

type	x	g(x)
endpoint	-3	4
critical	-1	0
critical	4	6
critical	6	4
"endpoint"	∞	-∞

h(x) on (-∞,∞)

type	x	h(x)
"endpoint"	-∞	0
critical	3	5
critical	4	3
critical	7	-4
"endpoint"	∞	0

F(x) on (-∞,∞)

type	x	F(x)
"endpoint"	-∞	∞
critical	-1	-2
critical	1	2
"endpoint"	∞	-∞

Sec 4.2,4.3,4.6: Examples of max and min

f(x) on [1,7]

type	x	f(x)	
endpoint	1	8	GLOBAL MAX
critical	3	-1	GLOBAL MIN
critical	6	6	
endpoint	7	5	

g(x) on [-3,∞)

type	x	g(x)	
endpoint	-3	4	
critical	-1	0	
critical	4	6	GLOBAL MAX
critical	6	4	
"endpoint"	∞	-∞	NO GLOBAL MIN

h(x) on (-∞,∞)

type	x	h(x)	
"endpoint"	-∞	0	
critical	3	5	GLOBAL MAX
critical	4	3	
critical	7	-4	GLOBAL MIN
"endpoint"	∞	0	

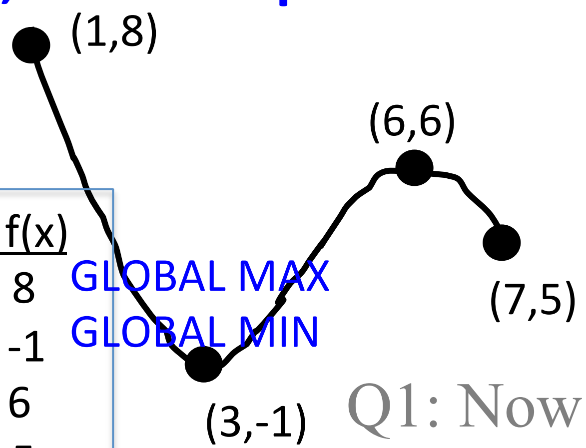
F(x) on (-∞,∞)

type	x	F(x)	
"endpoint"	-∞	∞	NO G MAX
critical	-1	-2	
critical	1	2	
"endpoint"	∞	-∞	NO G MIN

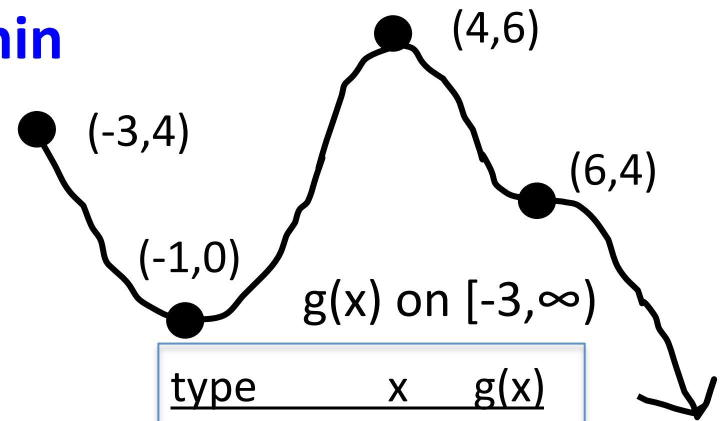
Sec 4.2,4.3,4.6: Examples of max and min

f(x) on [1,7]

type	x	f(x)	
endpoint	1	8	GLOBAL MAX
critical	3	-1	GLOBAL MIN
critical	6	6	
endpoint	7	5	

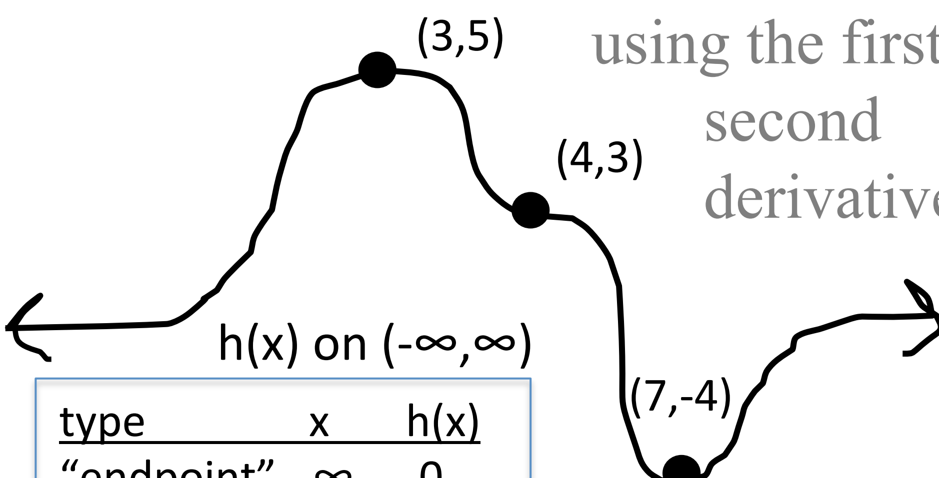


Q1: Now find the local maximums and local minimums using the first or second derivative test.



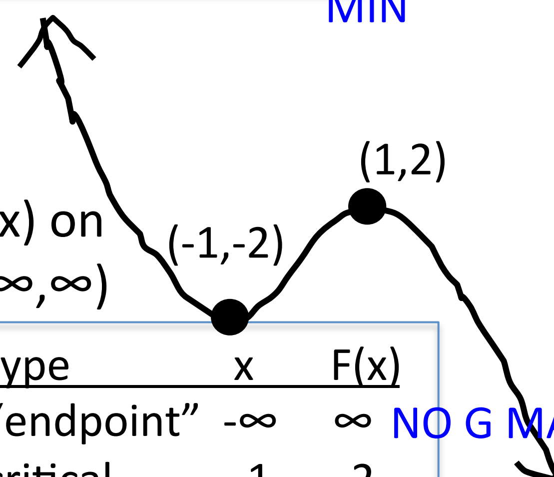
g(x) on [-3,∞)

type	x	g(x)	
endpoint	-3	4	
critical	-1	0	
critical	4	6	GLOBAL MAX
critical	6	4	
"endpoint"	∞	-∞	NO GLOBAL MIN



h(x) on (-∞,∞)

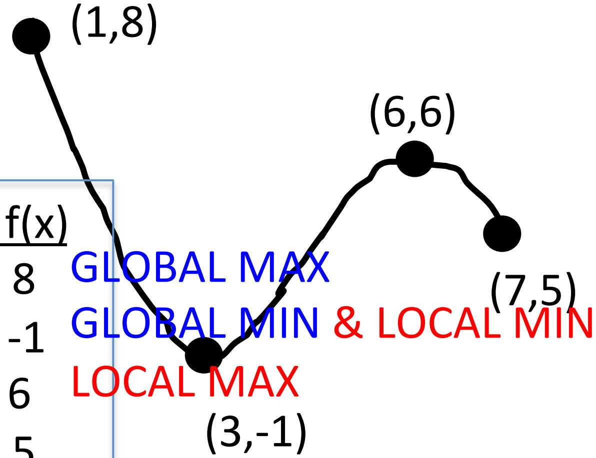
type	x	h(x)	
"endpoint"	-∞	0	
critical	3	5	GLOBAL MAX
critical	4	3	
critical	7	-4	GLOBAL MIN
"endpoint"	∞	0	



F(x) on (-∞,∞)

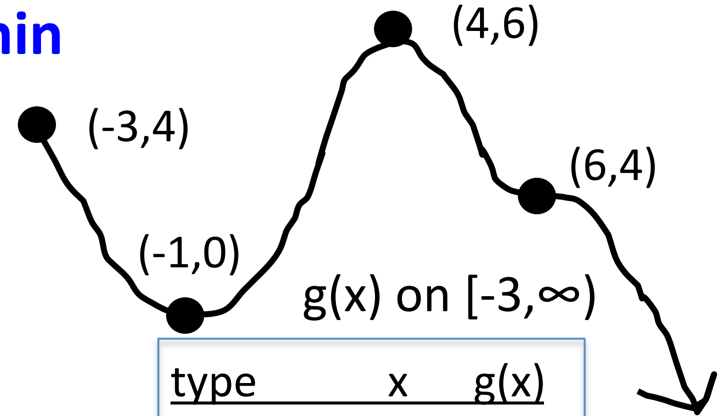
type	x	F(x)	
"endpoint"	-∞	∞	NO G MAX
critical	-1	-2	
critical	1	2	
"endpoint"	∞	-∞	NO G MIN

Sec 4.2,4.3,4.6: Examples of max and min



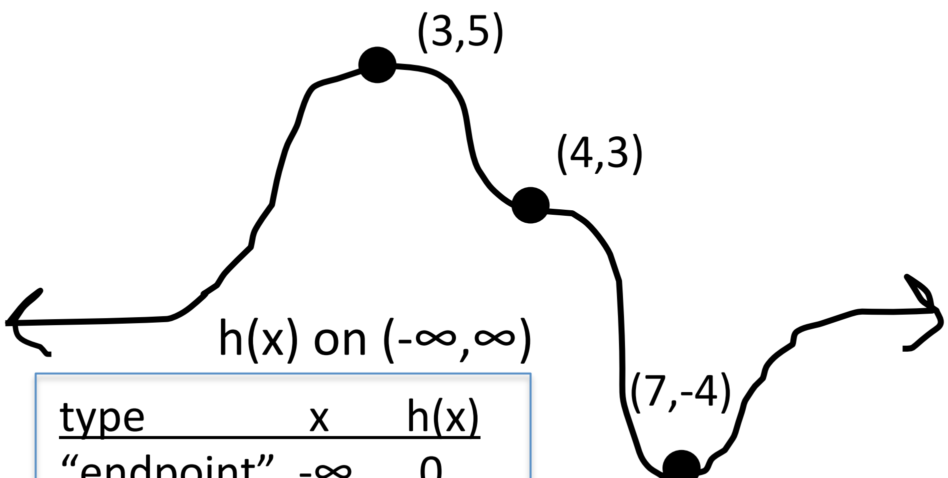
f(x) on [1,7]

type	x	f(x)	
endpoint	1	8	GLOBAL MAX
critical	3	-1	GLOBAL MIN & LOCAL MIN
critical	6	6	LOCAL MAX
endpoint	7	5	



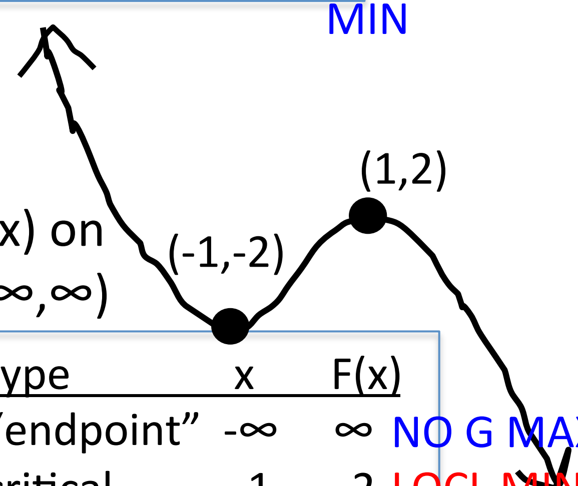
g(x) on [-3,∞)

type	x	g(x)	
endpoint	-3	4	
critical	-1	0	LOCAL MIN
critical	4	6	GLOBAL MAX & LCL MX
critical	6	4	
“endpoint”	∞	-∞	NO GLOBAL MIN



h(x) on (-∞,∞)

type	x	h(x)	
“endpoint”	-∞	0	
critical	3	5	GLOBAL MAX & LOCAL MAX
critical	4	3	
critical	7	-4	GLOBAL MIN & LOCAL MIN
“endpoint”	∞	0	



F(x) on (-∞,∞)

type	x	F(x)	
“endpoint”	-∞	∞	NO G MAX
critical	-1	-2	LOCL MIN
critical	1	2	LOCL MAX
“endpoint”	∞	-∞	NO G MIN

Sec 3.5-3.7: Derivative techniques

If x is in base and exponent, use logarithmic differentiation
If y cannot be solved for, use implicit differentiation

1. Compute the derivative of $f(x) = \arccos(e^{2x})$ on $x \leq 0$

2. Compute the derivative of $g(x) = \ln(\sin x)$ on $0 < x < \pi$

3. Compute the derivative of $h(x) = x^{\cos x}$

4. Find the equation to the tangent line to the curve $x^2y^2 + xy = 2$ at the point (1,1).

Sec 4.5: Limit techniques

If x is in base and exponent, use logarithmic limit technique
Else try plug in, and then L'Hopital rule or our old rules

5. Compute the following limits or state DNE.

(a) $\lim_{x\to\infty} x^{\frac{1}{x}}$ (b) $\lim_{t\to 0} \frac{t^3}{e^t - 1}$ (c) $\lim_{x\to\infty} \frac{(x+1)^5}{2x^5 - x}$

Sec 3.5-3.7: Solutions to Derivative techniques

1. $f'(x) = \dfrac{-2e^{2x}}{\sqrt{1-e^{4x}}}$ on $x \leq 0$

2. $g'(x) = \cot x$ on $0 < x < \pi$

3. $h'(x) = x^{\cos x}\left(-\sin x \ln x + \frac{\cos x}{x}\right)$ *via logarithmic differentiation*

4. Tangent line is $y = -\frac{3}{2}x + \frac{1}{2}$ *via implicit differentiation*

Sec 4.5: Solutions to Limit techniques

5. (a) $\lim_{x\to\infty} x^{\frac{1}{x}}$ =1 *via logarithmic limit*   (b) $\lim_{t\to 0} \frac{t^3}{e^t - 1}$ =0 *via L'Hopital's Rule*   (c) $\lim_{x\to\infty} \frac{(x+1)^5}{2x^5 - x}$ =1/2 *via our old rules*

Final Exam Old Topics (Midterm 1 and 2 stuff)

1.1-1.6 Precalculus

(a) basic function stuff

2.1-2.5 Limits

(a) tangent lines (b) derivative limit definition

(c) plug in, factor/reduce, rationalize, squeeze, asymptotes

(d) limits involving infinity

2.6-3.4 Derivatives

(a) constant, power, exponential, trig function rules

(b) sum, difference, product, quotient rules (c) chain rule

2.4, 2.7 Continuity and Differentiability

(a) when is a function continuous and differentiable

2.1-3.4 Examples of limit and derivative problems

1. Compute the following limits or state DNE.

(a) $\displaystyle\lim_{x\to 1}\frac{x^2-6x+8}{x^2+2x+1}$ (b) $\displaystyle\lim_{x\to\infty}\frac{\sqrt{x^4+x}}{3x^2-2x-1}$ (c) $\displaystyle\lim_{x\to-\infty}\frac{x}{\cos x}$

2. Let *u*(*x*)=*fg*(*x*), *v*(*x*)=(*f*/*g*)(*x*), and *w*(*x*)=(*f*∘*g*)(*x*).
Calculate the following.
u(3) *u'*(3)
v(2) *v'*(2)
w(1) *w'*(1)

x	f(x)	f'(x)	g(x)	g'(x)
1	1	8	2	7
2	-6	3	3	-1
3	-2	4	7	1

3. Compute the following derivatives

(a) $f(x)=\sin(x^3-x)$ (b) $g(x)=e^{5x}\cos(2x)$ (c) $h(x)=e^{\sin^2 x}$

2.1-3.4 Solutions of limit and derivative problems

1.
(a) limit = 3/4
(b) limit = 1/3
(c) limit = $-\infty$ DNE

2.
$u(3) = -14$ $u'(3) = 26$
$v(2) = -2$ $v'(2) = 1/3$
$w(1) = -6$ $w'(1) = 21$

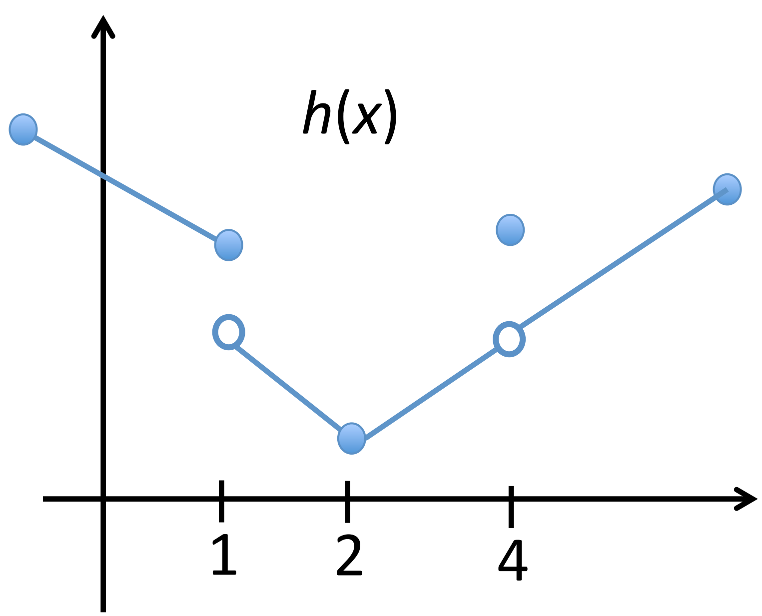
3.
(a) $f'(x) = (3x^2 - 1)\cos(x^3 - x)$
(b) $g'(x) = -2e^{5x}\sin(2x) + 5e^{5x}\cos(2x)$
(c) $h'(x) = e^{\sin^2 x}(2\sin x)\cos x$

Sec 2.4, 2.7 Examples of continuity and differentiability problems

1. For which value(s) of *a* is *f*(*x*) continuous?
2. For which value(s) of *b* is *g*(*x*) differentiable?

$$f(x) = \begin{cases} x^2 + 1 \text{ if } x \leq 1 \\ -ax^2 \text{ if } x > 1 \end{cases} \qquad g(x) = \begin{cases} e^x \text{ if } x < 0 \\ bx + 1 \text{ if } x \geq 0 \end{cases}$$

3. For each of the values *c* = 1, 2, 4, answer the following three questions. (a) Does $\lim_{x \to c} h(x)$ exist? (b) Is *h*(*x*) continuous at *x*=*c*? (c) Is *h*(*x*) differentiable at *x*=*c*?



Sec 2.4, 2.7 Solutions of continuity and differentiability problems

1. a = -2
2. b = 1
3. For c = 1, (a) no (b) no (c) no
   c = 2, (a) yes (b) yes (c) no
   c = 4, (a) yes (b) no (c) no