Math 10A MIDTERM #2 is in **Peter 108 at 8-9pm** next Wed, Nov 14

Log into TritonEd to view your assigned seat.

Midterm covers Sections 2.5-2.8, 3.1-3.4

You don't need blue books. Calculators are not allowed. You are allowed one double sided 8.5 by 11 inch page of handwritten notes. Bring your student ID.

Midterm 2 Topics

2.5 Limits involving infinity

(a) vertical asymptotes, horizontal asymptotes

Example: Find v and h asymptotes of $f(x) = \frac{4x^2 - x + 3}{x^2 - 3x + 2}$

2.6-2.7 Derivatives using limits

(a) slope of tangent, derivative at x=a, derivative function

Example: Find derivative of $f(x) = \frac{1}{x}$ using limits.

2.8 What do derivatives tell us?

(a) increasing, decreasing, local min & max, concave up, concave down, inflection point.

3.1-3.4 Derivatives using rules

(a) constant, power, exponential, trig function rules

(b) sum, difference, product, quotient rules (c) chain rule

Example: Find derivative of $f(x) = 3x^2 e^x + \cos^6 x$ using rules.

2.5-2.8, 3.1-3.4 Answers: Topic examples

Example 2.5. Vertical asymptotes are x=1 and x=2. Horizontal asymptotes are y=4.

Example 2.6-2.7. From limits, $f'(x) = -x^{-2}$

Example 3.1-3.4. $f'(x) = 3x^2e^x + 6xe^x – 6 (\cos^5 x) (\sin x)$

2.5 Review: Limits involving infinity

Suppose $$f(x) = \frac{b_n x^n + b_{n-1}x^{n-1} + ...b_2x^2 + b_1x + b_0}{c_m x^m + c_{m-1}x^{m-1} + ... + c_2x^2 + c_1x + c_0}$$

Then vertical asymptotes are at *x*=*d* for every *d* that makes the denominator equal zero.

And horizontal asymptotes exist only if *n*<=*m* where *n* is numerator degree and *m* is denominator degree

Case 1: *n* > *m* then $\lim_{x \to \infty} f(x) = \pm\infty$

Case 2: *n* < *m* then $\lim_{x \to \infty} f(x) = 0$ and *y* = 0 is horizontal asymptote

Case 3: *n* = *m* then $\lim_{x \to \infty} f(x) = \frac{b_n}{c_m}$ and $y = \frac{b_n}{c_m}$ is horizon. asymptote

2.5 Practice Questions: Limits involving infinity

1. Calculate the following limits or state that they do not exist (DNE). Also state if the limit approaches $+\infty$ or $-\infty$.

a. $\lim\limits_{x\to 2^+} \frac{3}{x-2}$ b. $\lim\limits_{x\to 2^-} \frac{3}{x-2}$ c. $\lim\limits_{x\to -\infty} \frac{3}{x-2}$

d. $\lim\limits_{x\to \infty} \frac{\sqrt{x^4-x}}{2x^2+7}$ e. $\lim\limits_{x\to \infty} \frac{x^4}{x^2-x+1}$

2. Find all vertical and horizontal asymptotes of the following functions

a. $f(x) = \frac{2x^2}{x^2-2x-8}$

b. $g(x) = \frac{4}{(x-1)(x+2)(x-3)}$

2.5 Answers: Limits involving infinity

1a. DNE, limit = $+\infty$

1b. DNE, limit = $-\infty$

1c. limit = 0

1d. limit = ½

1e. DNE, limit = $+\infty$

2a. Vertical asymptotes x=4, x=-2. Horizontal y=2

2b. Vertical asymptotes x=1, x=-2, x=3. Horizontal y=0.

2.6-2.7 Review: Derivatives using limits

The tangent slope of *f*(*x*) at *x*=*a*. **Below is a number**.

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The derivative of *f*(*x*) at *x*=*a*. **Below is a number**.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

The derivative of *f*(*x*) for all *x*. **Below is a function**.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

NOTE: The tangent slope of *f*(*x*) at *x*=*a* equals the derivative of *f*(*x*) at *x*=*a*. Therefore all the formulas above are equivalent.

2.6-2.7 Practice Questions: Derivatives using limits

1. Calculate the derivative of $f(x) = \sqrt{x}$ using a limit definition.

2. Calculate the derivative of $g(x) = x^2$ using a limit definition

3. Find the equation of the tangent line to the function $f(x) = e^x$ at *x*=0. Use derivative rules instead of limits.

4. Find the equation of the tangent line to the function $g(x) = x^3 - 4x^2 + 1$ at *x*=1. Use derivative rules instead of limits.

2.6-2.7 Answers: Derivatives using limits

1. From limits, $f'(x) = (1/2)x^{(-1/2)}$
2. From limits, $f'(x) = 2x$
3. $y=x+1$
4. $y= -5x+3$

2.8 Review: What do derivatives tell us?

A function is **increasing** when $f'(x)>0$.

A function is **decreasing** when $f'(x)<0$.

A function has a **local maximum** at $x=b$ if $f'(b)=0$
and $f'(a)>0$ and $f'(c)<0$ for $a<b<c$ with a and c close to b.

A function has a **local minimum** at $x=b$ if $f'(b)=0$
and $f'(a)<0$ and $f'(c)>0$ for $a<b<c$ with a and c close to b.

A function is **concave up** when $f''(x)>0$.

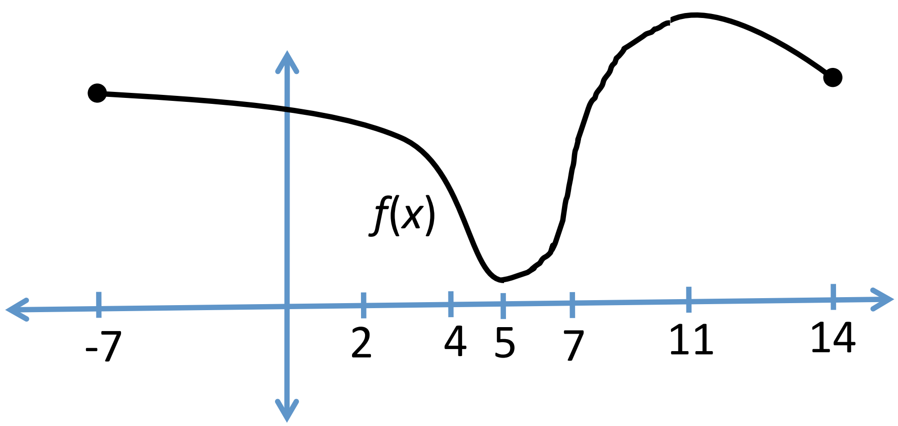
A function is **concave down** when $f''(x)<0$.

A function has an **inflection point** at $x=b$ if $f''(a)<0$ and $f''(c)>0$ for
$a<b<c$ with a and c close to b. Or if $f''(a)>0$ and $f''(c)<0$.

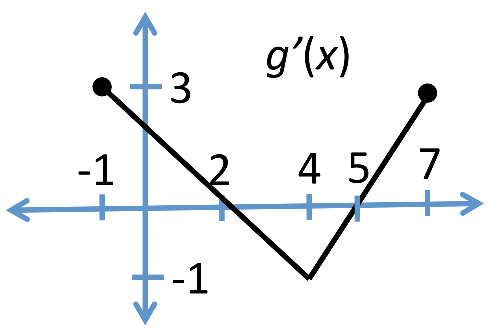
2.8 Practice Questions: What do derivatives tell us?

1. Draw a function h(x) that has a local min at x=1, local max at x=5, and inflection point at x=4.

2. For f(x) below, identify what intervals it is increasing, decreasing, concave up, and concave down. Next identify all local min, local max, and inflection points.



3. Regarding g(x), identify what intervals it is increasing, decreasing, concave up, and concave down. Next identify all local min, local max, and inflection points. The derivative of g(x) is drawn below.



2.8 Answers: What do derivatives tell us?

f(x) is increasing on (5,11)
f(x) is decreasing on (-7,5) and (11,14)
f(x) is concave up on (4,7)
f(x) is concave down on (-7,4) and (7,14)
f(x) has local min at x=5
f(x) has local max at x=11
f(x) has inflection points at x=4 and x=7

g(x) is increasing on (-1,2) and (5,7)
g(x) is decreasing on (2,5)
g(x) is concave up on (4,7)
g(x) is concave down on (-1,4)
g(x) has local min at x=5
g(x) has local max at x=2
g(x) has inflection point at x=4

3.1-3.4 Review: Derivatives using rules

constant function rule $\frac{d}{dx}c = 0$

power function rule $\frac{d}{dx}x^n = nx^{n-1}$

exponential function rule $\frac{d}{dx}e^x = e^x$

trig function rules $\frac{d}{dx}sin(x) = cos(x)$ $\quad \frac{d}{dx}cos(x) = -sin(x)$

constant multiplier rule $\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}f(x)$

sum rule $\frac{d}{dx}\left[f(x) + g(x)\right] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

difference rule $\frac{d}{dx}\left[f(x) - g(x)\right] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

product rule $\frac{d}{dx}\left[(fg)(x)\right] = f(x)g'(x) + f'(x)g(x)$

quotient rule

$$\frac{d}{dx}\left[\frac{f}{g}(x)\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

chain rule $\frac{d}{dx}[(f \circ g)(x))] = \frac{d}{dx}[f(g(x))] = f'\big(g(x)\big) \cdot g'(x)$

3.1-3.4 Practice Questions: Derivatives using rules

1. Let $u(x)=fg(x)$, $v(x)=(f/g)(x)$, and $w(x)=(f \circ g)(x)$. Calculate the following six values.

$u(1)$ $u'(2)$

$v(2)$ $v'(0)$

$w(1)$ $w'(0)$

x	f(x)	f'(x)	g(x)	g'(x)
0	5	8	2	9
1	-3	-7	1	3
2	4	-2	4	6

2. Calculate the derivatives of the following six functions

$$F(x) = \sqrt{\sin(3x)} \qquad G(x) = \frac{e^x}{x+1} \qquad H(x) = e^{(x^3+x)}$$

$$U(x) = x^2\cos(e^x) \qquad V(x) = (x^3-1)^7 \qquad W(x) = 2\sin x\cos x$$

$$\frac{d}{dx}\left[\frac{f}{g}(x)\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2} \qquad \frac{d}{dx}[(fg)(x)] = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx}[(f \circ g)(x))] = \frac{d}{dx}[f(g(x))] = f'\big(g(x)\big) \cdot g'(x)$$

3.1-3.4 Answers: Derivatives using rules

1. u(1) = -3, u'(2)=16,
   v(2)=1, v'(2)= -2,
   w(1)= -3, w'(0)= -18

2. $F'(x) = \frac{3\cos(3x)}{2\sqrt{\sin(3x)}}$ $G'(x) = \frac{xe^x}{(x+1)^2}$

   $H'(x) = (3x^2+1)e^{(x^3+x)}$

   $U'(x) = -x^2 e^x \sin(e^x) + 2x\cos(e^x)$

   $V'(x) = 21x^2(x^3-1)^6$

   $W'(x) = -2\sin^2 x + 2\cos^2 x$

$$\frac{d}{dx}\left[\frac{f}{g}(x)\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}[(fg)(x)] = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx}[(f\circ g)(x))] = \frac{d}{dx}[f(g(x))] = f'\big(g(x)\big)\cdot g'(x)$$