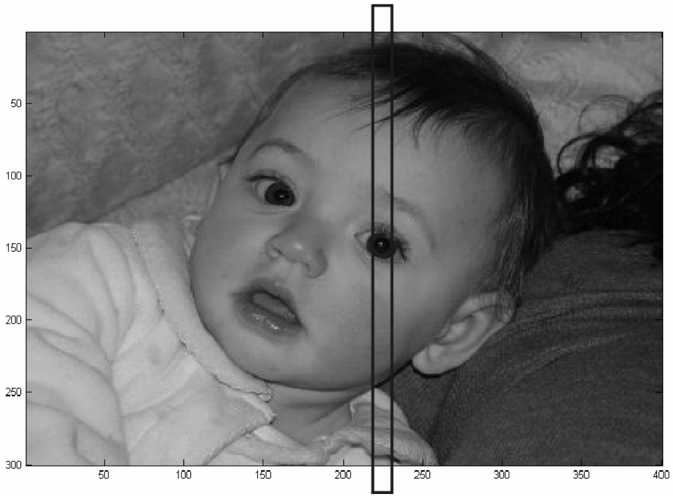
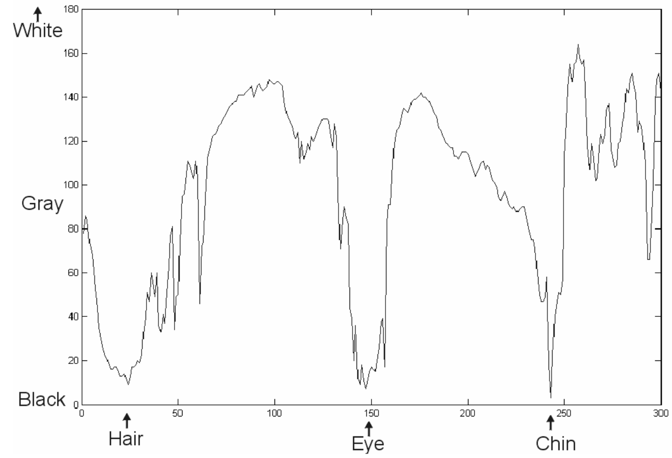
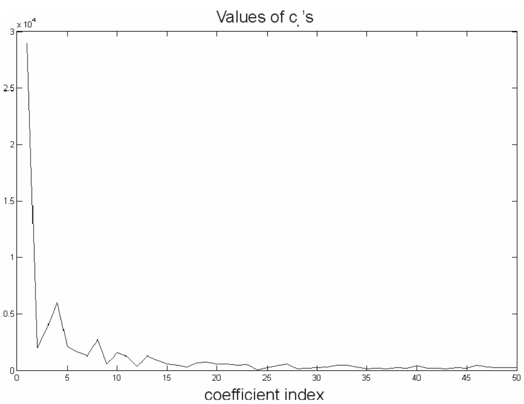




The column of the image inside the rectangle above is 300 pixels tall (also pretend it is one pixel wide). If you denote black=0, white=255, and the shades of gray as continuous values between 0 and 255, you can represent this column as the discrete function pictured above and to the right. Let's call this function $f(y)$ for $y \in [0,299] \subset \mathbb{R}$ alternatively you can think of this column of pixels as a vector $x \in \mathbb{R}^{300}$. In either case, we can apply the Discrete Fourier Transform, $c = \Omega x$ with $\Omega \in \mathbb{R}^{300 \times 300}$ where $\Omega_{i,j} = e^{-\left(\frac{2\pi}{300}\right)(i-1)(j-1)}$, thus $f(y) \sim \frac{1}{300}\sum_{k=0}^{299} c_{k+1} e^{\left(\frac{2\pi}{300}\right)iky} = \frac{1}{300}c_1 e^0 + \frac{1}{300}c_2 e^{\left(\frac{2\pi}{300}\right)iy} + \frac{1}{300}c_3 e^{\left(\frac{2\pi}{300}\right)2iy} + \ldots + \frac{1}{300}c_{299} e^{\left(\frac{2\pi}{300}\right)298iy} + \frac{1}{300}c_{300} e^{\left(\frac{2\pi}{300}\right)299iy}$.

Looking at the coefficients, c_k, we see that for larger values of *k*, the coefficients become smaller and less important. This is what we would expect. The *c*'s with larger indices are coefficients of exponential functions with a shorter period and hence "wiggle faster". Images don't tend to have fast fluctuations from the color of one pixel to its neighbor's pixel color. Therefore, we can retain most of the original column information by using only the first *r* terms of the Fourier expansion.



Let $r = 20$ then $\widetilde{f}(y) = \frac{1}{300}\sum_{k=0}^{19} c_{k+1} e^{\left(\frac{2\pi}{300}\right)iky}$. This is the same discrete function as $\widetilde{f}(y) = \left[\Omega^{-1} d \in \mathbb{R}^{300}\right]_{y+1}$ where $d = \left[c(1:20)^T \quad 0\ 0\ 0\ \ldots\ 0\right]^T \in \mathbb{R}^{300}$, $c = \Omega x$.

Therefore instead of storing this column as 300 numbers, we can save only 20 numbers, c_1 to c_{20}, and still have a decent approximation. That amounts to a file size that is $\frac{20}{300} \approx 6.6\%$ of the original. If we do this to every column, than the original image which required 120,000 bytes to store can now be stored in 8,000. Below are the approximated image and $\widetilde{f}(x)$ for $x \in [0,299]$.