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THE DIPOLE OSCILLATIONS OF GENERAL RELATIVISTIC NEUTRON STARS

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ABSTRACT

We investigate the theory of the dipole oscillations of fully relativistic neutron stars. The equations governing these modes are reduced to a third-order system; and the variational principle for the frequencies of these modes is reduced to a form involving only two independent functions. The equations that determine the damping of these modes, which is due to viscous dissipation, are presented. The frequencies and viscous damping times for a range of realistic neutron star models based on a number of equations of state and a range of masses are determined by solving these dipole oscillation equations numerically.

Subject headings: relativity — stars: neutron — stars: pulsation

I. INTRODUCTION

The dipole oscillations of nonrotating general relativistic stellar models are the only nonradial modes that do not couple to gravitational radiation. Probably for this reason, these modes have received little attention in the literature. The differential equations describing the dipole modes were derived by Campolattaro and Thorne (1970), and criteria for their stability were found by Detweiler (1975) using a variational principle for their frequencies. These equations have never been solved (to our knowledge) except in the "relativistic Cowling approximation" by McDermott, Van Horn, and Scholl (1983). One of the motivations for the present work is to solve the complete equations for the dipole *p*-modes of realistic neutron-star models numerically.

The dissipative effects of gravitational radiation and viscosity play an interesting and important role in the nonradial *f*- and *p*-modes of neutron stars. While these effects only damp out the oscillations in nonrotating stars, they also drive the instabilities that limit the angular velocities of rotating neutron stars (see, e.g., Chandrasekhar 1970; Lindblom and Detweiler 1977; Friedman 1983; Lindblom 1986, 1987, 1988). The angular velocity dependence of the frequencies of the dipole modes has never been computed because these modes do not exist at all in the only well-studied rotating stellar models: the Maclaurin spheroids. Therefore, it is not clear at this time whether or not a secular instability can exist in the dipole modes of realistic rotating stars. It is interesting to note, however, that the viscosity-driven secular instability in the $l = -m$ modes occurs at lower angular velocities for lower values of l (for $l \geq 2$ where it has been studied). It is possible, therefore, that a dipole mode viscosity-driven secular instability could exist in some rotating stellar models, and it is conceivable that the instability could be the dominant one (i.e., setting in at the lowest angular velocity) in some stars as well. It will be necessary to await the results of the computations of the angular velocity dependence of the frequencies and viscous damping times for these modes under way by Ipser and Lindblom before we will know for sure whether or not dipole mode secular instabilities exist. In this paper we lay the foundation for that work by determining the effects of viscosity on the dipole *p*-modes of realistic, fully relativistic nonrotating neutron star models.

Section II of this paper describes the mathematical formalism needed to describe the dipole oscillations of nonrotating general relativistic stellar models. We reduce the fourth-order system of equations for the dipole oscillations given by Campolattaro and Thorne (1970) to a third-order system. We also show how the variational principle for the frequencies of these modes given by Detweiler (1975) can be written in a form that depends on only two of the perturbation functions. Finally, we derive the equations for the viscous damping times of these dipole oscillations. Section III describes the numerical algorithm that we use to solve the dipole oscillation equations, and the results of our computations for a number of neutron star models based on a variety of nuclear matter equations of state and a range of neutron star masses are presented. We also show (as a check of the accuracy of our computations) that the frequencies of a sequence of fully relativistic polytropic stellar models does correctly approach the corresponding dipole frequency of the analogous Newtonian stellar model.

II. THEORY OF DIPOLE OSCILLATIONS

This section presents the mathematical formalism needed to analyze the dipole oscillations of fully general relativistic stellar models. We treat the oscillations here as infinitesimal linear perturbations of nonrotating (spherical) stars. The background spherical stellar models are briefly discussed; then a discussion of the adiabatic (i.e., nondissipative dipole pulsation equations is presented which is simpler than previously published accounts; finally, we present the equations needed to analyze the damping of these modes by viscosity. (Gravitational radiation damping does not occur in the dipole oscillations of nonrotating stars.)

a) Background Stellar Models

The gravitational field of a static spherical star in general relativity theory is given by the metric tensor

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2) , \qquad (1)$$

where ν and λ are functions of r only. These functions are determined, along with the variables describing the physical state of the fluid in the star, ρ and p (the total energy density and pressure, respectively), from Einstein's equation:

$$\lambda' = \frac{1 - e^{\lambda}}{r} + 8\pi r e^{\lambda} \rho , \qquad (2)$$

$$\nu' = -\lambda' + 8\pi r e^{\lambda}(\rho + p) , \qquad (3)$$

$$p' = -\tfrac{1}{2} (\rho + p)\nu' , \qquad (4)$$

where the prime denotes differentiation with respect to r, and we have set $G = c = 1$. Given an equation of state $\rho = \rho(p)$, the numerical integration of these equations (or other well-known equivalent forms) is a long-studied, and well-understood subject (see, e.g., Oppenheimer and Volkoff 1939; Arnett and Bowers 1977). We comment on only one nuance of the analysis. The surface of the star is located by determining the radius R at which the pressure falls to zero: $p(R) = 0$. In general, however, the pressure does not go to zero linearly in r, making the zero difficult to locate. This problem has been solved by inserting many numerical steps into the integration near the surface of the star. In the analysis of the oscillations of stars, however, we find it to be more convenient to work on a fixed uniformly spaced radial grid. It is still possible, however, to locate the surfaces of our models accurately by considering the thermodynamic enthalpy density,

$$e(p) = \int_0^p \frac{d\pi}{\rho(\pi) + \pi}, \tag{5}$$

of the star. The surface of the star is also a zero of the enthalpy density: $e[p(R)] = 0$. Since equation (4) is equivalent to $\nu' = -2e'$, and since ν' does not vanish on the surface of a star, it follows that e goes linearly to zero. It is straight forward to determine the location of such a zero accurately.

b) *Adiabatic Dipole Oscillations*

The state of a stellar model perturbed away from its equilibrium configuration may be described in terms of the Lagrangian displacement vector ξ^a and the perturbed metric tensor δg_{ab}. For the perturbations that represent the dipole oscillations of a star, it is convenient to make gauge choices (see Campolattaro and Thorne 1970) so that ξ^a is represented as

$$\xi_a = \left[e^{\lambda/2}\left(\frac{W}{r^2}\right)Y^1{}_m \nabla_a r + V\nabla_a Y^1{}_m\right]e^{i\omega t}, \tag{6}$$

where V and W are functions of r only, $Y^1{}_m$ is one of the standard $l = 1$ (dipole) spherical harmonics, ω is the constant representing the frequency of the mode, and λ is the metric function from the background stellar model (eqs. [1]–[4]). Similarly, the perturbed metric, δg_{ab}, can be represented in a suitably chosen gauge as

$$\delta g_{ab}\, dx^a dx^b = (H_0 e^\nu dt^2 + 2i\omega H_1\, dt\, dr + H_2 e^\lambda\, dr^2)Y^1{}_m e^{i\omega t}, \tag{7}$$

where H_0, H_1, and H_2 are functions of r only. Einstein's equation, linearized about a fixed static spherical background, reduces to a real system of ordinary differential equations for the five functions V, W, H_0, H_1, and H_2 as first shown by Campolattaro and Thorne (1970). Their equations are equivalent to the following system:

$$16\pi r^2\omega^2(\rho + p)e^{\lambda-\nu}V = 8\pi(\rho + p)\nu' e^{\lambda/2}W + (\nu' - 2r\omega^2 e^{-\nu})H_1 + (16\pi r^2 \rho e^\lambda - 3r\lambda')H_0, \tag{8}$$

$$rH_2 = -H_1 - 8\pi(\rho + p)e^{\lambda/2}W, \tag{9}$$

$$8\pi p\gamma e^{\lambda/2}W' = (\tfrac{1}{2}r\nu' - 4\pi p\gamma r^2 e^\lambda)H_2 + (1 - e^\lambda - \tfrac{1}{2}r\nu')H_0 + r\omega^2 e^{-\nu}H_1 - 8\pi p' e^{\lambda/2}W + 16\pi p\gamma e^\lambda V, \tag{10}$$

$$rH_0' = (1 - \tfrac{1}{2}r\nu')H_0 - (1 + \tfrac{1}{2}r\nu')H_2 - r\omega^2 e^{-\nu}H_1, \tag{11}$$

$$rH_1' = [\tfrac{1}{2}r(\lambda' - \nu') - e^\lambda]H_1 - 8\pi(\rho + p)e^{3\lambda/2}W - 16\pi r(\rho + p)e^\lambda V. \tag{12}$$

(We note that these equations and the other complicated algebraic expressions which appear in this paper were derived, manipulated, and simplified using the tensor algebraic computer language muTENSOR developed by J. F. Harper and C. C. Dyer.) In these equations γ is the adiabatic index which we compute directly from the equilibrium structure of the star as

$$\gamma = \frac{\rho + p}{p}\frac{p'}{\rho'}. \tag{13}$$

The first two of the perturbation equations (8) and (9) are algebraic constraints which can be used to determine V and H_2 in terms of the remaining functions: W, H_0, and H_1. The remaining three equations (10)–(12) form a third-order system of ordinary differential equations for the three functions W, H_0, and H_1. We note that the algebraic condition in equation (8) was (to our knowledge) previously unknown, and its discovery allowed the reduction of the system of equations from the fourth-order system, presented by Campolattaro and Thorne (1970), to the third-order system used here.

The system of equations (10)–(12), together with appropriate boundary conditions at the center of the star ($r = 0$) and the surface of the star ($r = R$), form an eigenvalue problem for the dipole modes. The boundary conditions at the surface of the star require that the gravitational potentials H_0 and H_1 vanish there:

$$H_0(R) = H_1(R) = 0. \tag{14}$$

At the center of the star, the boundary conditions are based on the requirement that the physical perturbation variables (i.e., ξ^a, δg_{ab}, $\delta\rho$) are finite. These conditions require that

$$\lim_{r\to 0}\left(\frac{W}{r^2}\right) = w, \tag{15}$$

$$\lim_{r\to 0}\left(\frac{H_0}{r}\right) = h, \tag{16}$$

$$\lim_{r\to 0}\left(\frac{H_1}{r^2}\right) = -8\pi(\rho_c + p_c)w, \tag{17}$$

where h and w are constants not fixed by the boundary conditions, and ρ_c and p_c are the values of the density and pressure at the center of the star.

c) *Variational Principle for the Frequency*

It is often extremely helpful to have available a variational principle from which the frequency can be computed. We use the variational principle described here to obtain initial estimates of the frequency of a mode, and, once we have obtained a solution to the mode equations described above, we use the variational principle as a redundant check on the value (and hence the accuracy) of the frequency. The variational principle can also be used to estimate the frequencies of these modes without an exact knowledge of the eigenfunctions using a Rayleigh-Ritz technique.

A variational principle for the frequencies of these modes was first derived by Detweiler (1975). His variational principle is equivalent to

$$\omega^2 \int_0^R e^{-\nu/2}\left[(\rho + p)e^{\lambda/2}\left(\frac{W^2}{r^2} + 2V^2\right) - e^{-\lambda/2}\frac{(H_1)^2}{8\pi}\right]dr = \int_0^R e^{\nu/2}\left\{r^2 e^{\lambda/2} p\gamma\left[\frac{\delta\rho}{\rho + p}\right]^2 - \frac{1}{16\pi}(1 + r\nu')e^{-\lambda/2}(H_2)^2\right\}dr + \frac{\rho(R)\, MW^2(R)}{R^4} \tag{18}$$

for the case considered here, where γ is given by equation (13) (and up to a typographical error in Detweiler's paper which has been corrected here). The constant M which appears in equation (18) is the total mass of the star, while $\rho(R)$ is the value of the density at the surface of the star (i.e., possibly zero depending upon the equation of state).

We also point out that this variational principle can be written in a way that depends only on the two functions W and H_1. The functions V and H_2 that appear in equation (18) can be replaced by expressions obtained from equations (12) and (9), respectively, which only depend on W and H_1. Similarly, the "Eulerian" perturbation in the density, $\delta\rho$, which appears in equation (18), is given by

$$\delta\rho = -\frac{e^{-\lambda/2}}{r^2}\left[(\rho + p)W' + \rho' W + (\rho + p)e^{\lambda/2}(\tfrac{1}{2}r^2 H_2 - 2V)\right], \tag{19}$$

which depends only on W and H_1 when the expressions for V and H_2 from equations (12) and (9) are used. This form of the variational principle (which involves only W and H_1) has one useful and one interesting consequence. The useful consequence of this form would be realized if it were used to estimate the frequencies with a Rayleigh-Ritz technique; it would only require the use of two parameterized test eigenfunctions. The interesting consequence of this form of the variational principle is that it implies that the mode equations (10)–(12) could be rewritten as a set of two coupled second-order equations for the functions W and H_1 alone. (To obtain these second-order equations one would simply vary this form of the variational principle with respect to W and H_1.) We chose to perform our numerical integration of these modes using the third-order system, equations (10)–(12), rather than "reducing" the equations in this way to a fourth-order system in W and H_1 alone.

d) *Viscous Dissipation*

While the time-scale for the damping of the dipole modes by viscosity is very brief by astronomical standards ($\sim 10^4$ s), it is, in fact, very long compared to the dynamical time-scale of the oscillation. The viscosity is in effect, then, a very weak force which may be treated as a small modification of the adiabatic oscillations discussed above. One effect of this small viscosity will be to introduce a small imaginary part to the frequency of the mode, $\omega \to \omega + i/\tau$, which will damp out the oscillation. This viscous damping time, τ, may be computed by evaluating the rate at which the energy of the mode is dissipated by the viscosity. Lindblom and Hiscock (1983) have shown that the energy contained in a mode, $E(t)$, evolves in the presence of viscosity as

$$\frac{dE}{dt} = -\int 2\eta\, \delta\sigma_{ab}{}^{*}\, \delta\sigma^{ab} e^{\nu}\, d^3x\,, \tag{20}$$

where η is the viscosity of the star, $\delta\,\sigma^{ab}$ is the shear of the velocity perturbation, and dx^3 is the proper three-volume element on a $t =$ constant hypersurface. Since this energy is quadratic in the perturbation functions, the time derivative of the energy is also given by

$$\frac{dE}{dt} = -\frac{2E}{\tau}\,, \tag{21}$$

for a perturbation with time dependence $e^{i\omega t - t/\tau}$. By eliminating dE/dt from equations (20) and (21), an expression for the viscous damping time, τ, is obtained in terms of the energy, E, and a integral involving the shear, $\delta\sigma_{ab}$, of the pulsations. Lindblom and Hiscock (1983) have given expressions for these quantities in terms of ξ^a and δg_{ab}. Therefore, using the parameterization of the dipole modes given in equations (6) and (7), it is straightforward to obtain the following representations of these quantities (which are valid to lowest order in the viscosity):

$$E = \int_0^R \omega^2\left[(\rho + p)e^{(\lambda-\nu)/2}\left(\frac{W^2}{r^2} + 2V^2 + e^{-\lambda/2} W H_1\right) - e^{-(\lambda+\nu)/2} r\,\frac{H_1 H_2}{8\pi}\right] dr\,, \tag{22}$$

$$\int 2\eta\delta\sigma^{*}{}_{ab}\,\delta\sigma^{ab} e^{\nu} d^3x = 2\omega^2 \int_0^R \eta(6\alpha_1{}^2 + 4\alpha_2{}^2) r^2 e^{\lambda/2}\, dr\,, \tag{23}$$

where α_1 and α_2 are given by

$$\alpha_1 = \frac{H_2}{6} + \frac{V}{3r^2} + \frac{e^{-\lambda/2}}{r^2}\left(\frac{W'}{3} - \frac{W}{r}\right), \tag{24}$$

$$\alpha_2 = \frac{W}{2r^3} + \frac{e^{-\lambda/2}}{r}\left(\frac{V'}{2} - \frac{V}{r}\right). \tag{25}$$

To lowest order in the viscosity the eigenfunctions and frequencies that appear in these expressions are the same as those for the adiabatic oscillations discussed above. Thus the viscous damping time is given approximately by

$$\frac{1}{\tau} = \frac{\omega^2}{E}\int_0^R \eta(6\alpha_1{}^2 + 4\alpha_2{}^2) r^2 e^{\lambda/2}\, dr\,, \tag{26}$$

where E is to be interpreted as the integral in equation (22). These integrals are straightforward to evaluate numerically once the eigenfunctions for the adiabatic oscillations are known.

III. NUMERICAL EVALUATION OF THE DIPOLE OSCILLATIONS

In this section, the methods and the results of our numerical evaluation of the dipole oscillation equations are discussed. Equations (10)–(12), together with the boundary conditions equations (14)–(17), constitute a two-point boundary eigenvalue problem. We solve this system of equations numerically using a "shooting" technique (see, e.g., Press *et al.* 1986, p. 586) as follows. We begin by making a rough guess of the eigenfunction corresponding to the lowest frequency dipole mode (the p_1 mode). (In practice we find the following to be reasonable initial guesses: $H_1 = -8\pi(\rho_c + p_c)r^2[1 - (r/R)^2]$ and $W = r^2[1 - (2r/R)^6]$.) These test eigenfunctions are used with the variational principle (eq. [18]) to obtain an initial estimate of the frequency ω. Next we make guesses of the values of the constants h and w that appear in the boundary conditions, equations (15)–(17). (A bit of trial and error is needed to find these.) Once these initial choices for h, w, and ω are made, the oscillation equations (10)–(12) can be integrated away from the center of the star to a matching point at some $r = r_0 < R$. Next we choose a value for the constant $W(R)$ and integrate the equations (10)–(12) with the boundary conditions given by equation (14) starting at the surface of the star ($r = R$) back to the matching point at $r = r_0$. If the two sets of values for $W(r_0)$, $H_0(r_0)$, and $H_1(r_0)$ so obtained fail to agree, we use their discontinuity to adjust and improve the values of the constants h, w, $W(R)$, and ω, by Newton's method. Using these improved constants, the equations are reintegrated from both boundaries to the matching point at $r = r_0$. This process is iterated until the values of these constants converge, and the desired degree of continuity of the functions W, H_0, and H_1 at the matching point is achieved.

We have evaluated the accuracy of the frequencies that we obtain using the algorithm described above in three different ways. First, we verified that as we increased the number of numerical grid points in our models, the difference between the frequency of a given model and the most accurate frequency computed was proportional to $1/K^2$, where K is the total number of grid points in the model. This dependence of the accuracy with the number of grid points is consistent with the accuracy with which we impose the boundary conditions on the pulsations, for example. The second test of the accuracy of our algorithm involved the comparison of the frequencies computed directly (using the shooting technique discussed above) with frequencies obtained from the variational principle (eq. [18]) using the directly computed values for the eigenfunctions. In all of our models these two independently computed values of the frequencies differ by less than 0.1%. The third and most stringent test of the accuracy of our program involved the comparsion of our frequencies with previously published results. The only published value (that we know about) of an exact calculation of a dipole oscillation frequency (using an adiabatic index computed as in eq. [13]) is contained in the paper of Hurley, Roberts, and Wright (1966). There they obtain the value $\omega = 2.9696\ (3GM/4R^3)^{1/2}$ for the frequency of the dipole p_1-mode of an $n = 3/2$ polytrope (i.e., for the equation of state $p = \kappa\rho^{5/3}$, where κ is a constant) using the Newtonian stellar pulsation equations. For Newtonian polytropes the ratio $\omega/(3GM/4R^3)^{1/2}$ is independent of the constant κ and the central density of the star ρ_c. Since our relativistic dipole oscillation equations should reduce to the Newtonian equations for stars of sufficiently small mass, we can verify that our frequencies do reduce to this value in the appropriate limit. Figure 1 shows the results of our computations of the dipole frequencies for fully relativistic stars having the equation of state $p = \kappa\rho^{5/3}$ and $\kappa = 8.675 \times 10^9$ cm^2 g$^{-2/3}$ s^{-2}. Unlike their Newtonian counterparts, however, the relativistic frequencies do depend on the parameters of a particular stellar model, like GM/c^2R. Figure 1 illustrates that our computations of the relativistic dipole oscillations do approach the correct Newtonian value in the limit that $GM/c^2R \rightarrow 0$.

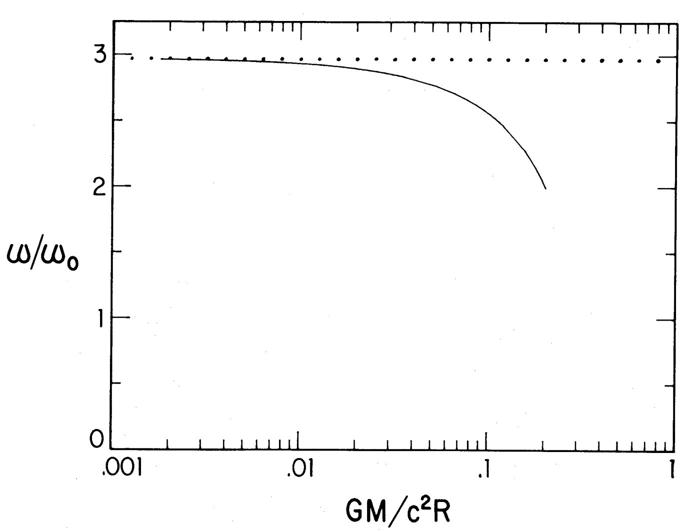

FIG. 1.—The frequencies of the dipole modes of $n = 3/2$ polytropes are given in units of $\omega_0 = (3GM/4R^3)^{1/2}$. The solid line gives the frequencies as a function of GM/c^2R for a sequence of fully general relativistic stellar models, while the dotted curve gives the frequency of the corresponding models computed using Newtonian equations by Hurley, Roberts, and Wright (1965).

We have computed the frequencies and eigenfunctions for a range of neutron star models based on a variety of nuclear density equations of state and for a range of neutron star masses. The frequencies and the viscous damping times for the lowest frequency dipole mode (having one node in each of the eigenfunctions V and W) are reported in Table 1. The equations of state used here are the standard ones discussed in the literature as described in detail in Baym and Pethick (1979) or Shapiro and Teukolsky (1983), p. 228, for example. (Except for the RMF equation of state where we use the Serot 1979 calculation as tabulated in Lindblom and Detwiler 1983.) For each equation of state we evaluate the frequencies for three different models: the maximum mass model, the model having a mass of 1.4 $M_\odot$ (the only neutron star mass actually observed), and the model having a total baryon number of 1.4 $N_\odot$ (where $N_\odot$ is the number of baryons in the Sun; this is approximately the minimum number of baryons that can collapse to form a neutron star). The viscous damping times that are reported in Table 1 were evaluated using equations (22)–(26). For the viscosity of neutron star matter we use the expression,

$$\eta = 6.0 \times 10^6 \rho^2/T^2\ , \tag{27}$$

where T is the temperature of the matter, and all quantities are expressed in cgs units. This expression (given in Cutler and Lindblom 1987) approximates the electron-electron scattering viscosity that is appropriate for neutron star matter which is cooler than the superfluid critical temperature, $T_c \approx 10^9$ K. While the ratio τ/τ_0 is independent of temperature, the product $\tau\omega_0$ (a measure of the damping time in units of the dynamical time scale of the oscillations, i.e., a measure of the Q of the oscillations) is proportional to the temperature squared. The values of this quantity given in Table 1 correspond to a neutron star central temperature of 10^7 K (the star is assumed to be "isothermal" in the relativistic sense that $Te^{\nu/2}$ is constant). We note that the frequencies listed in Table 1 verify that ω and τ are reasonably well (i.e., to within a factor of ~ 2) approximated by the rough estimates $\omega_0 = (\pi G\rho_{\rm ave})^{1/2}$ and $\tau_0 = R^2\rho_{\rm ave}/\eta(\rho_{\rm ave})$, where $\rho_{\rm ave} = 3M/4\pi R^3$. The extremely large values of $\tau\omega_0$ also verify our contention that the viscosity in these models is effectively very small and that justifies the small viscosity approximations that were made to obtain an expression (eq. [26]) for τ.

Figure 2 illustrates typical examples of the eigenfunctions W, H_0, and H_1 for the dipole p_1-mode. The particular functions represented here were computed for the 1.4 $M_\odot$ stellar model based on equation of state R. These exact eigenfunctions show that the fluid motion in these modes (as determined by the function W) is confined to the surface region of the star as anticipated by the work by McDermott, Van Horn and Scholl (1983) in the Cowling approximation. Figure 3 illustrates another aspect of the eigenfunctions of this same mode: the energy dissipation rate due to viscous interaction. This figure graphs the energy dissipation rate (the integrand in eq. [26]) as a function of the density ρ. This illustrates that even though the eigenfunction W is strongly peaked at the surface, some important dynamical quantities have support much deeper into the

TABLE 1

PROPERTIES OF DIPOLE OSCILLATIONS OF NEUTRON STARS

Equation of State	ρ_c (g cm^{-3})	$M/M_\odot$	$N/N_\odot$	$1/\omega_0$[a] (ms)	ω/ω_0	τ/τ_0[b]	$\tau\omega_0$[c]
π	2.666×10^{15}	1.243	1.399	0.0683	2.872	1.091	1.83×10^7
	3.447×10^{15}	1.400	1.617	0.0595	2.563	1.822	2.40×10^7
	5.550×10^{15}	1.483	1.742	0.0511	2.081	3.138	3.00×10^7
R	1.659×10^{15}	1.267	1.399	0.0896	2.947	1.373	4.39×10^7
	1.977×10^{15}	1.400	1.570	0.0814	2.750	1.655	4.52×10^7
	4.100×10^{15}	1.623	1.881	0.0610	2.064	3.004	4.62×10^7
BJ	0.974×10^{15}	1.300	1.399	0.1280	2.912	1.107	8.28×10^7
	1.102×10^{15}	1.400	1.520	0.1189	2.822	1.212	8.01×10^7
	3.150×10^{15}	1.850	2.110	0.0733	2.044	2.693	6.93×10^7
TI	0.447×10^{15}	1.314	1.399	0.1772	3.518	0.940	1.51×10^8
	0.519×10^{15}	1.400	1.498	0.1696	3.370	1.020	9.45×10^7
	2.239×10^{15}	1.759	1.935	0.0981	2.042	1.889	9.29×10^7
RMF	0.504×10^{15}	1.292	1.400	0.1389	4.221	1.014	9.14×10^7
	0.535×10^{15}	1.400	1.528	0.1347	4.071	1.151	1.02×10^8
	2.000×10^{15}	2.563	3.081	0.0847	1.908	4.068	1.82×10^8
MF	0.418×10^{15}	1.304	1.399	0.1603	4.088	0.807	1.02×10^8
	0.440×10^{15}	1.400	1.511	0.1554	3.991	0.902	1.11×10^8
	1.500×10^{15}	2.661	3.157	0.0976	2.106	3.439	2.21×10^8

[a] $\omega_0 \equiv (\pi G \rho_{avg})^{1/2}$, where $\rho_{avg} = 3M/4\pi R^3$.

[b] $\tau_0 \equiv R^2 \rho_{avg}/\eta(\rho_{avg})$.

[c] Damping times are given for neutron star central temperatures of $T = 10^7$ K.

interior of the star. We have plotted the energy dissipation rate as a function of the density of the stellar material to illustrate that the majority of the viscous dissipation occurs at densities above 10^{14} g cm^{-3}. The viscosity given in equation (27) should be the appropriate one for cool neutron stars in this density range. The discontinuities that appear in Figure 3 are caused by discontinuities in the adiabatic index, γ, which in turn are a result of the standard method used to interpolate the equation of state tables (see, Arnett and Bowers 1977).

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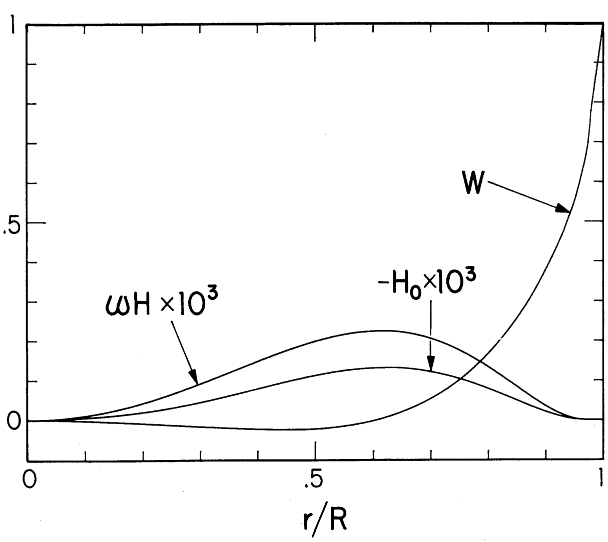

FIG. 2

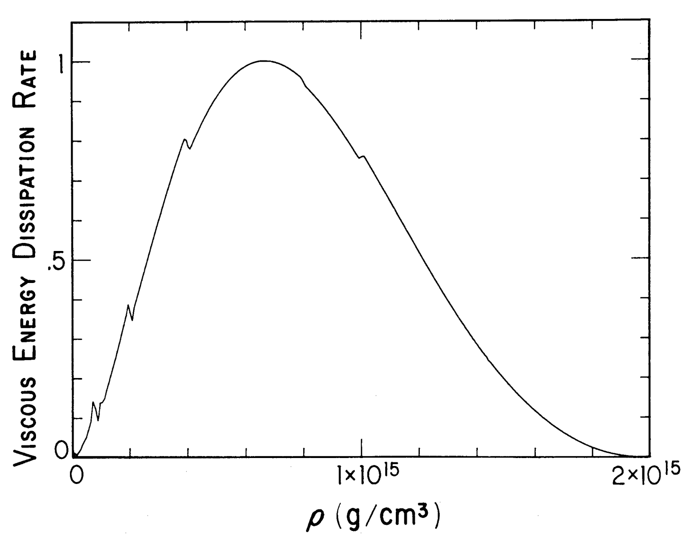

FIG. 3

FIG. 2.—The basic eigenfunctions for the lowest frequency dipole p-mode of the 1.4 $M_\odot$ neutron star model based on equation of state R. The function W is normalized so that $W(R) = 1$, while the normalization for the metric perturbations H_0 and ωH_1 relative to W is determined by the geometrical units used in our computations (in which $G = c = 1$, and all lengths are expressed in kilometers).

FIG. 3.—The energy dissipation rate which is due to viscosity (i.e., the integrand in eq. [23]) in the lowest frequency dipole p-mode of the 1.4 $M_\odot$ neutron star model based on equation of state R. This quantity is graphed as a function of the matter density, ρ, in this particular stellar model, and it is normalized so that its maximum value is one.

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