GRAVITATIONAL RADIATION FROM INSTABILITIES IN RAPIDLY ROTATING NEUTRON STARS?

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Summary. Rapidly rotating neutron stars are known to be subject to an instability driven by gravitational radiation[1-4]. The modes responsible for this instability are non-axisymmetric, having azimuthal angular dependence $\exp\{im\phi\}$ with typical values of m in the range[5-7] $2 \leq m \leq 5$. These modes are also strongly damped by the presence of viscosity, since the fluid motion contained in them is strongly sheared[8-10]. If the viscosity of neutron star matter is sufficiently large, therefore, the instability in these modes can be completely suppressed by viscous damping and the emission of significant amounts of gravitational radiation would be blocked. The viscosity of neutron star matter varies with temperature[11,10] as T^{-2}, thus sufficiently cold neutron stars will have large viscosities that suppress the instabilities in these modes. The critical temperature below which these modes are completely stabilized has been estimated[10,12] to be $T \approx 10^7$ K. This is close to the maximum temperature to which a neutron star can be heated by accretion. Therefore, it is not clear at this time whether it is possible for rapidly rotating neutron stars spun up by accretion to undergo this instability and emit significant amounts of gravitational radiation.

This brief report summarizes the method used to estimate the properties of the modes that are responsible for limiting the angular velocity of neutron stars. The calculation of the temperature dependence of the critical angular velocities of these modes will be outlined. The oscillation frequencies of these modes for stars rotating at the critical angular velocity are computed here (for the first time). These are the frequencies at which any gravitational radiation would be emitted or the frequencies at which the x-ray flux might be modulated by a star spun up by accretion to the point of one of these non-axisymmetric instabilities. The oscillation periods for the relevant modes are in the range .01 to .001 seconds. A more accurate computation of these effects, including the temperature dependence of the critical angular velocity of this instability and the expected temperatures of accreting neutron stars, are needed before it will be possible to predict with confidence whether or not this kind of system is likely to be an observationally interesting source of gravitational radiation.

Finding the Critical Angular Velocities. The time dependence of the non-axisymmetric mode that limits the angular velocity, Ω, of a rotating neutron star will in general have the form:

$$\exp\{i\sigma_m(\Omega)t - t/\tau_m(\Omega)\}.$$

The real part of the frequency, $\sigma_m(\Omega)$, governs the oscillation rate of the star while the imaginary part of the frequency, $1/\tau_m(\Omega)$, determines the rate at which the mode is damped (or amplified) by the dissipative effects of gravitational radiation reaction and viscosity. This mode becomes unstable at the critical angular velocity, Ω_m, where the imaginary part of the frequency changes sign: i.e., where $0 = 1/\tau_m(\Omega_m)$.

To aid in the calculation of these critical angular velocities it is helpful to define the dimensionless functions $\alpha_m(\Omega)$, $\beta_m(\Omega)$ and $\gamma_m(\Omega)$ by the equations:

$$\sigma_m(\Omega) = \sigma_m(0)\, \alpha_m(\Omega) - m\,\Omega, \tag{1}$$

$$\frac{1}{\tau_m(\Omega)} = \beta_m(\Omega) \left[\tau^{-1}_{V,m}(0) + \tau^{-1}_{GR,m}(0) \left[\frac{\sigma_m(\Omega)}{\sigma_m(0)\gamma_m(\Omega)}\right]^{2m+1} \right]. \tag{2}$$

In these equations $\tau_{V,m}(0)$ and $\tau_{GR,m}(0)$ represent the timescales for damping this mode due to viscosity and gravitational radiation reaction in the non-rotating star of the same rest mass. The function $\alpha_m(\Omega)$, therefore, describes the angular velocity dependence of the real part of the frequency; $\beta_m(\Omega)$ describes the angular velocity dependence of the viscous damping of the mode; and $\gamma_m(\Omega)$ describes the relative effects of viscosity and gravitational radiation reaction on the mode.

A useful expression for the critical angular velocity, Ω_m, of a mode can be inferred from eq. (2):

$$\Omega_m = \frac{\sigma_m(0)}{m}\left[\alpha_m(\Omega_m) + \gamma_m(\Omega_m)\left[\frac{\tau_{GR,m}(0)}{\tau_{V,m}(0)}\right]^{1/(2m+1)}\right]. \qquad (3)$$

The solutions to eq. (3) are, quite generally, the critical angular velocities of these modes in any rotating stellar model. These equations are useful, however, only after the values of the relevant frequencies of these modes in the associated non-rotating stellar model [$\sigma_m(0)$, $\tau_{GR,m}(0)$, and $\tau_{V,m}(0)$] and the functions α_m, β_m and γ_m are known. While the properties of non-rotating stars are relatively easy to compute, the functions that describe the angular velocity dependence of the modes can, unfortunately, be determined only with great difficulty. To date, the functions α_m have been computed for the Maclaurin spheroids[7] and for a few rigidly rotating Newtonian polytropes[13]; the functions β_m and γ_m have only been computed for the Maclaurin spheroids[7]. None of these functions varies significantly from its non-rotating value, $\alpha_m(0) = \beta_m(0) = \gamma_m(0) = 1$, over the entire range of relevant angular velocities in the Maclaurin spheroids. Furthermore, the functions α_m computed for the Newtonian polytropes are very similar to the corresponding functions computed for the Maclaurin spheroids. It seems likely, therefore, that the functions α_m, β_m, γ_m will not depend strongly on the equation of state of the stellar model.

Since the functions that describe the angular velocity dependence of these modes are not expected to depend strongly on the equation of state, it is possible to find approximate solutions to eq. (3) for the locations of the critical angular velocities: Use the Maclaurin spheroid functions α_m and γ_m together with the appropriate values of $\sigma_m(0)$, $\tau_{GR,m}(0)$ and $\tau_{V,m}(0)$ computed using fully relativistic equations and realistic models of the stellar material[7,10]. While the accuracy of this method will only be known after the α_m, β_m and γ_m are computed properly for realistic neutron star models (work now in progress in collaboration with J. Ipser), it seems likely that the accuracy is better than 30% which is the maximum deviation of the functions α_m and γ_m from their non-rotating values of one.

The Temperature Dependence of Ω_c. In the temperature range below about 10^9 K the neutrons and protons in the core of a neutron star are expected to exist in a superfluid state. The viscosity of this material is expected, therefore, to be dominated by electron-electron scattering[11,14]. For the density range in which most of the material in a neutron star is found, the following simple analytic expression,

$$\eta = 6.0 \times 10^6 \rho^2 T^{-2},$$

gives the electron-electron viscosity to about 5% accuracy. Since this expression (and indeed the "exact" expresson on which it is based) varies with temperature as T^{-2}, the viscous timescale $\tau_{V,m}(0)$ that appears in eq. (3) will vary as T^2. Therefore, the critical angular velocities, Ω_c, that are the solutions of eq. (3) will depend on the temperature of the neutron star.

The approximation method described above has been used to compute the critical angular velocities for a variety of neutron star models over a range of temperatures[7,10,12]. Figure 1 depicts the temperature dependence of the critical angular velocities for two neutron star

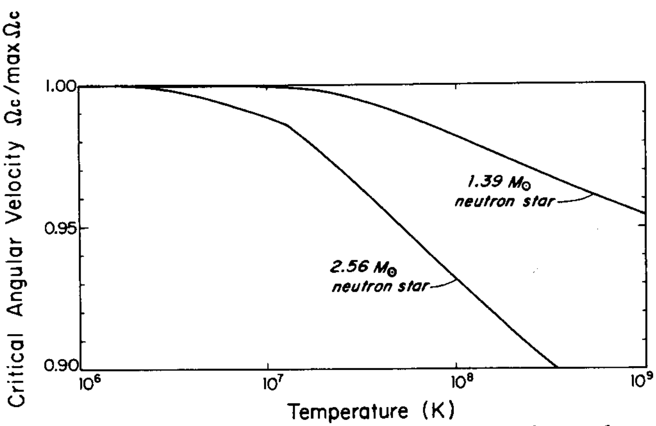

Figure 1. The temperature dependence of the critical angular velocities for the gravitational radiation reaction secular instability.

models. The angular velocities are expressed as a fraction of the maximum angular velocity that a Maclaurin spheroid can attain [max $\Omega = 0.670322(\pi G\bar{\rho})^{1/2}$]. (This maximum angular velocity agrees with those computed for realistic fully relativistic stellar models[15] to within about 5%.) The discontinuities in the derivatives of the curves in Figure 1 are the location of the points where the mode responsible for the instability is changing from m = 2 at low temperature to m = 3 at higher temperatures. The two stellar models depicted in this graph are the maximum mass stellar model and the model containing 1.4 times the number of baryons in the sun (the minimum mass core that can collapse to form a neutron star) based on a self-consistent relativistic mean field model for the nuclear matter[16)]. This graph clearly illustrates that as the temperature of a neutron star drops, the increasing viscosity of neutron star matter is able to suppress the gravitational radiation driven secular instability so that stars with larger and larger angular velocities are stable. This graph also suggests that below about 10^6 to 10^7 K the instability is completely suppressed. Figure 2 examines in more detail the highest angular velocity portion of the critical angular velocity curves for a variety of different equations of state (labeled A, B, C etc. and

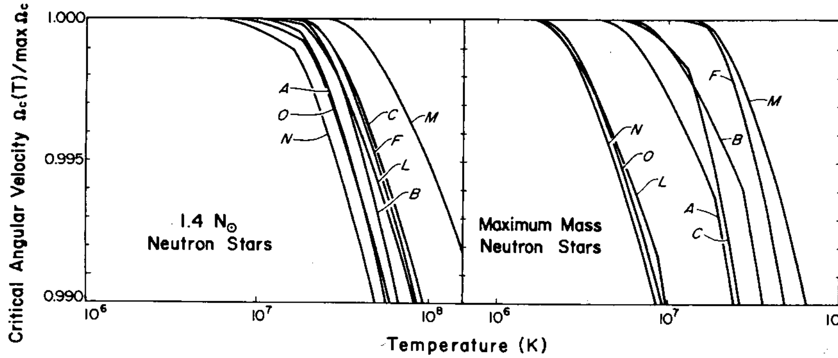

Figure 2. The temperature dependence of the critical angular velocities for a range of neutron star masses and a variety of equations of state for the high density nuclear matter.

defined in Ref. 7). No stellar model having less angular momentum than the model rotating with maximum angular velocity is subject to the gravitational radiation driven secular instability for temperatures below about 1.5 x 10^6 K. (For stars having 1.4 times the solar number of baryons this minimum temperature is about 5 x 10^6 K). The largest value of this minimum temperature is 7 x 10^7 K for the models studied. While the accuracy of the approximation methods used to compute these minimum temperatures is unknown, it is likely that the error is reasonably large. As Figure 1 illustrates, relatively small changes in the critical angular velocity computed for a given temperature could change the minimum temperature of this instability by large amounts. Even a 5% change in the angular velocity estimates could change the minimum temperature values by a factor of ten. Since the minimum temperatures computed here are fairly close to the maximum temperature of about 10^7 K to which a neutron star can be heated by accretion, it is not possible to predict at this time whether or not the gravitational radiation driven secular instability will occur in real astrophysical situations. More accurate models of these instabilities are currently under construction.

Frequencies of the Unstable Modes. If these instabilities do exist in real neutron stars at sufficiently low temperatures, it has been suggested[6)] that significant amounts of gravitational radiation could be emitted under suitable circumstances. Consider a neutron star spun up by accretion to the critical angular velocity of the gravitational radiation secular instability discussed here. As additional angular momentum is deposited onto the star by the accretion process the angular velocity would increase slightly into the unstable region and the instability would set in. The amplitude of the mode would grow until the angular momentum carried away by gravitational radiation balanced the angular momentum deposited onto the star by accretion. Under these circumstances nearly monochromatic gravitational radiation would be emitted at the frequency of the unstable mode of the star rotating at essentially the critical angular velocity. Furthermore, the surface of the star would be distorted by the mode. Consequently it is possible that the accretion onto this uneven surface could modulate the x-ray flux produced by the accreting material at this same frequency. (The amplitude of these surface modulations have been estimated and a manuscript is in preparation in collaboration with W. Kluzniak and R. Wagoner.) It is easy to estimate the oscillation frequency of one of these modes for any given angular velocity using eq. (1). Using the approximation method described above (i.e. Maclaurin spheroid functions α_m and fully relativistic $\sigma_m[0]$) these oscillation frequencies have been computed for a number of neutron star models. Figure 3 depicts the temperature dependence of the oscillation periods of these modes for the two stars considered in Figure 1.

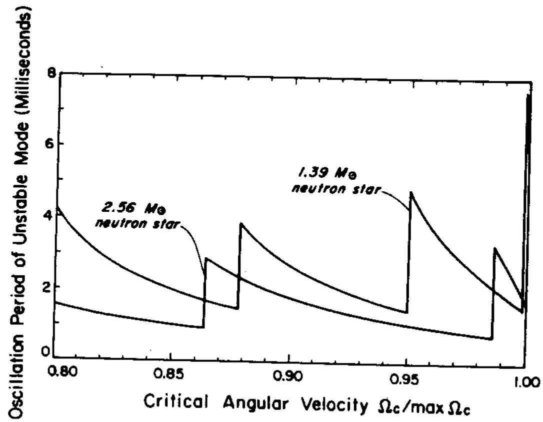

Figure 3. The temperature dependence of the oscillation periods for the modes that give rise to the gravitational radiation reaction secular instabilities.

These periods are in the range .01 to .001 seconds for all of the neutron stars examined. They are longer by about an order of magnitude than the period of the corresponding mode in the non-rotating star of the same rest mass. The discontinuities in the curves occur at the temperatures at which the mode responsible for the instability changes from one value of m to another.

This research was supported by NSF grant PHY-8518490.

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