Accuracy standards should be imposed on model waveform and detector calibration accuracies:

- to prevent a significant rate of missed detections,
- to prevent accuracy losses in measurements,
- to avoid unnecessary costs of achieving excess accuracy.

This talk will describe possible abuses of the standards, and ways to avoid them.
Waveform and Calibration Accuracy Standards:

- Combined waveform and calibration accuracy standards:

$$\sqrt{\langle \delta h_m | \delta h_m \rangle} + \sqrt{\langle \delta h_R | \delta h_R \rangle} < \left\{ \frac{1}{\rho \sqrt{2\epsilon_{\text{max}}}} \right\}$$

- $\delta h_m = h_m - h_e$  Model waveform error.
- $\delta h_R$  Errors from calibration inaccuracies.

- Standards are written in terms of the noise-weighted inner product:

$$\langle h_e | h_m \rangle = 2 \int_{0}^{\infty} \frac{h_e^*(f)h_m(f) + h_e(f)h_m^*(f)}{S_n(f)} df,$$

where $S_n(f)$ is the power spectral density of the detector noise.

- The maximum allowed errors are determined by $\rho$, the signal to noise ratio, and $\epsilon_{\text{max}}$ which determines the missed detection loss rate (typically set to $\epsilon_{\text{max}} = 0.005$).
Waveform accuracy standards can be re-written as:

\[ \frac{\sqrt{\langle \delta h_m|\delta h_m \rangle}}{\rho} = \sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \left\{ \begin{array}{ll}
1/(2\rho_{\text{max}}) & \text{measurement,} \\
\sqrt{2\epsilon_{\text{max}}} & \text{detection.}
\end{array} \right. \]

- Amplitude \( \delta \chi_m \) and phase \( \delta \Phi_m \) errors are defined as

\[ \delta h_m = h_e e^{i \delta \chi_m + i \delta \Phi_m} - h_e \approx h_e(\delta \chi_m + i \delta \Phi_m). \]

- Signal-weighted average errors are defined as

\[ \overline{\delta \chi_m^2} = \int_0^\infty \delta \chi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df, \quad \text{and} \quad \overline{\delta \Phi_m^2} = \int_0^\infty \delta \Phi_m^2 \frac{4|h_e|^2}{\rho^2 S_n} df. \]
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How do you relate \( \delta \chi_m(f) \) and \( \delta \Phi_m(f) \) to the time-domain waveform errors that arise in waveform modeling?

How do you estimate these errors reliably?
Maximum Error Fallacy

- Some NR groups have estimated the maximum time-domain waveform errors $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$, and compared them with the standards for $|\delta \chi_m|$ and $|\delta \Phi_m|$.
- Is this good enough?
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- Is this good enough?
- Consider a model waveform: $h_m(t)$ with errors of the form:

$$h_m(t) = A_e(t) \left[ 1 + \max|\delta \chi_t| g_\chi(t) \right] \cos \left[ \Phi_e(t) + \max|\delta \Phi_t| g_\Phi(t) \right],$$

with $g_\chi = g_\Phi = \cos[\lambda \Phi_e(t)].$
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with $g \chi = g \Phi = \cos[\lambda \Phi_e(t)]$.

- Compute ratio of frequency- to time-domain error measures,

$$ R = \sqrt{\frac{\delta \chi_m^2 + \delta \Phi_m^2}{\max(|\delta \chi_t|^2 + |\delta \Phi_t|^2)}} $$

using the PN+Caltech/Cornell waveform for $A_e$ and $\Phi_e$. 
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- Bad News! Limiting $\max|\delta \chi_t|$ and $\max|\delta \Phi_t|$ is not sufficient.
Error Envelope Fallacy

- Additional knowledge of the full waveform errors, \( \max |\delta \chi_t| g_\chi(t) \) and \( \max |\delta \Phi_t| g_\Phi(t) \), is needed. Unfortunately the exact time dependencies, \( g_\chi(t) \) and \( g_\Phi(t) \), will never be known.
- Is a partial knowledge of \( g_\chi(t) \) and \( g_\Phi(t) \) sufficient?
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- Is a partial knowledge of $g_\chi(t)$ and $g_\Phi(t)$ sufficient?

- Probably the most we will ever know will be local-in-time error envelope-functions $G_\chi(t)$ and $G_\Phi(t)$, that satisfy
  \[ |g_\chi(t)| \leq G_\chi(t) \leq 1, \quad \text{and} \quad |g_\Phi(t)| \leq G_\Phi(t) \leq 1. \]

- Do time-domain bounds imply frequency-domain bounds, i.e., does $|g(t)| \leq G(t)$ imply $|g(f)| \leq G(f)$? 
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- Do time-domain bounds imply frequency-domain bounds, i.e., does \( |g(t)| \leq G(t) \) imply \( |g(f)| \leq G(f) \)?

- No!

- It is not possible to verify the accuracy of a waveform using a time-domain error-envelope function.
An alternate form of the accuracy standards can be written in terms of the time domain $L^2$ norm $\|\delta h_m(t)\|^2 = \int_{-\infty}^{\infty} |\delta h_m|^2 dt$.

This alternate standard has the form:

$$\frac{\|\delta h(f)\|}{\|h_m(f)\|} = \frac{\|\delta h(t)\|}{\|h_m(t)\|} < \frac{C}{2\rho},$$

where $C$, is a scale invariant ratio of two signal-to-noise measures

$$C^2 = \frac{\rho^2}{2 \|h_m(f)\|^2 / \min S_n(f)} \leq 1.$$

The error envelope functions, $\max |\delta \chi_t| G_{\chi}(t)$ and $\max |\delta \Phi_t| G_{\Phi}(t)$, provide strict upper limits for these error measures.
Combined accuracy standards now exist for waveform accuracy and calibration. The model waveform standards can be written as:

$$\sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \left\{ \begin{array}{ll} \frac{1}{2\rho_{\text{max}}} & \text{measurement,} \\ \sqrt{2\epsilon_{\text{max}}} & \text{detection.} \end{array} \right.$$ 

The basic standards are difficult (impossible?) to enforce directly, so easier to enforce time-domain conditions have been derived:

$$\frac{\|\delta h_m(t)\|}{\|h_m(t)\|} \leq \sqrt{\int_{-\infty}^{\infty} A_m^2 \left( \max|\delta \chi_t|^2 G_{\chi}^2 + \max|\delta \Phi_t|^2 G_{\Phi}^2 \right) dt} \lesssim \left\{ \begin{array}{ll} C/(2\rho_{\text{max}}) & \text{} \\ C\sqrt{2\epsilon_{\text{max}}} & \text{} \end{array} \right.$$
Summary and Questions

- Combined accuracy standards now exist for waveform accuracy and calibration. The model waveform standards can be written as:

\[ \sqrt{\delta \chi_m^2 + \delta \Phi_m^2} < \begin{cases} \frac{1}{2 \rho_{\text{max}}} & \text{measurement,} \\ \sqrt{2 \epsilon_{\text{max}}} & \text{detection.} \end{cases} \]

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\[ \frac{\| \delta h_m(t) \|}{\| h_m(t) \|} \leq \sqrt{\int_{-\infty}^{\infty} A_m^2 \left( \max |\delta \chi_t|^2 G_{\chi}^2 + \max |\delta \Phi_t|^2 G_{\Phi}^2 \right) dt} \lessapprox \begin{cases} C/(2 \rho_{\text{max}}) & \text{C} \sqrt{2 \epsilon_{\text{max}}} \end{cases} \]

- How well do the calibration and search template accuracies currently being used by LIGO satisfy these requirements?

- How well do the waveforms produced by various NR groups satisfy these requirements?