Numerical Simulations of Black Hole Spacetimes

Lee Lindblom

Senior Research Associate
Theoretical Astrophysics

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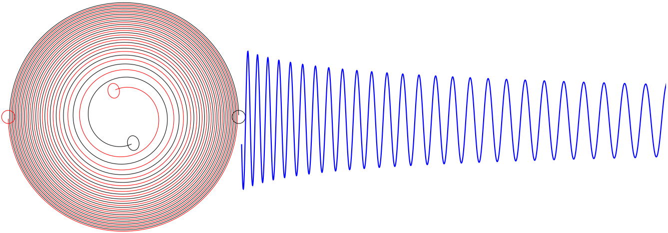

- Caltech-Cornell Numerical Relativity Collaboration

  Group leaders: Lee Lindblom, Mark Scheel, and Harald Pfeiffer at Caltech; Saul Teukolsky and Larry Kidder at Cornell.

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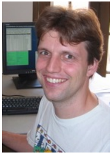

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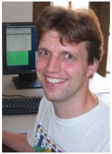
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- Caltech group: Michael Boyle, Jeandrew Brink, Duncan Brown, Tony Chu, Michael Cohen, Lee Lindblom, Geoffrey Lovelace, Keith Matthews, Robert Owen, Harald Pfeiffer, Oliver Rinne, Mark Scheel, Kip Thorne.
- Cornell group: Matthew Duez, Francois Foucart, Lawrence Kidder, Francois Limousin, Abdul Mroue, Nick Taylor, Saul Teukolsky, James York.

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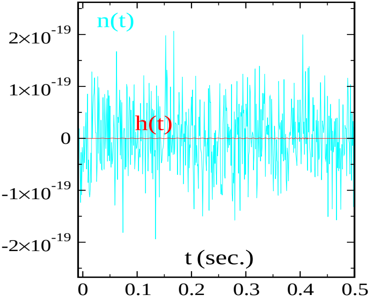

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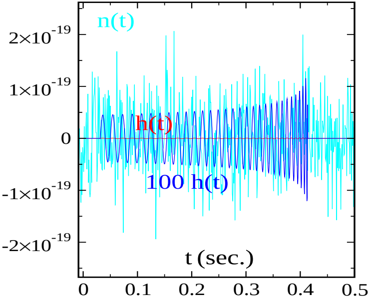

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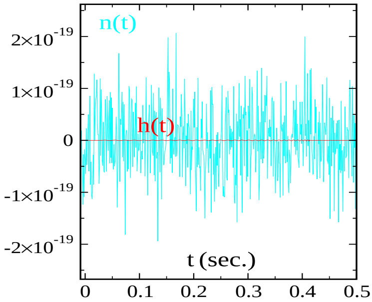

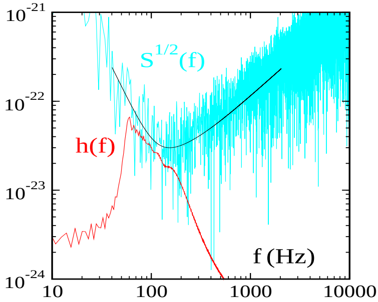

Why Is Numerical Relativity So Difficult?

- Dynamics of binary black hole problem is driven by delicate adjustments to orbit due to emission of gravitational waves.
- Very big computational problem:
  - Must evolve ~ 50 dynamical fields (spacetime metric plus all first derivatives).
  - Must accurately resolve features on many scales from black hole horizons $r \sim GM/c^2$ to emitted waves $r \sim 100GM/c^2$.
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  - Many grid points are required $\gtrsim 10^6$ even if points are located optimally.
- Most representations of the Einstein equations have mathematically ill-posed initial value problems.
- Constraint violating instabilities destroy stable numerical solutions in many well-posed forms of the equations.

Unstable BBH Movie

Recent Progress in Numerical Relativity

- Frans Pretorius performs first numerical BBH inspiral, merger and ringdown calculations in the spring of 2005 using a “generalized harmonic” formulation of the Einstein equations. Pretorius Inspiral Movie

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- Groups at NASA GSFC and U. Texas–Brownsville simultaneously announce similar BBH simulations in the fall of 2005 using very different methods (BSSN–puncture).
  LSU/AEI collaboration obtains similar results in Dec. 2005.
- Penn State group begins the study of physical properties of BBH orbits in early 2006 by evolving unequal mass binaries and measuring the kick velocity using BSSN–puncture methods.
- ...

Outline of Remainder of Talk:

- Three technical issues:
  - Constraint Damping.
  - Spectral Methods.
  - Feedback Control Systems.
- Interesting binary black hole simulations.

Gauge and Constraints in Electromagnetism

- The usual representation of the vacuum Maxwell equations split into evolution equations and constraints:

$$\partial_t \vec{E} = \vec{\nabla} \times \vec{B}, \qquad \nabla \cdot \vec{E} = 0,$$
$$\partial_t \vec{B} = -\vec{\nabla} \times \vec{E}, \qquad \nabla \cdot \vec{B} = 0.$$

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- This form of Maxwell's equations is manifestly hyperbolic as long as the gauge is chosen correctly, e.g., let $\nabla^a A_a = H(x,t)$, giving:

$$\nabla^a \nabla_a A_b \equiv \left(-\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2\right) A_b = \nabla_b H.$$

Constraint Damping

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- Modify evolution equations by adding multiples of the constraints:

$$\nabla^a\nabla_a A_b = \nabla_b H + \gamma_0 t_b \mathcal{C} = \nabla_b H + \gamma_0 t_b \left(\nabla^a A_a - H\right).$$

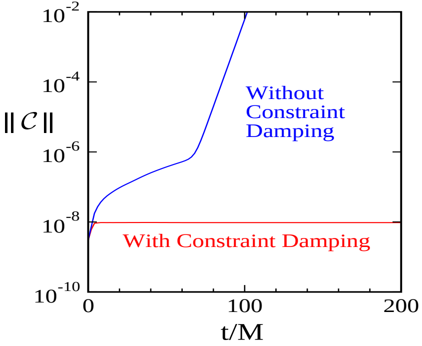
- These changes also affect the constraint evolution equation,

$$\nabla^a\nabla_a \mathcal{C} - \gamma_0 t^b \nabla_b \mathcal{C} = 0,$$

so constraint violations are damped when $\gamma_0 > 0$.

Constraint Damped Einstein System

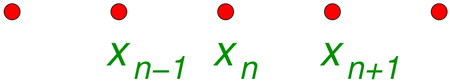
- "Generalized Harmonic" form of Einstein's equations have properties similar to Maxwell's equations:
  - Gauge (coordinate) conditions are imposed by specifying the divergence of the spacetime metric: $\partial_a g^{ab} = H^b + \ldots$
  - Evolution equations become manifestly hyperbolic: $\Box g_{ab} = \ldots$
  - Gauge conditions become constraints.
  - Constraint damping terms can be added which make numerical evolutions stable.



Numerical Solution of Evolution Equations

$$\partial_t u = F(u, \partial_x u, x, t).$$

- Choose a grid of spatial points, x_n.



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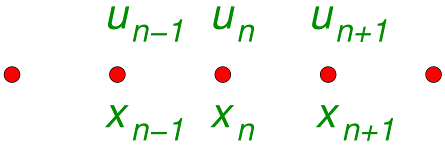

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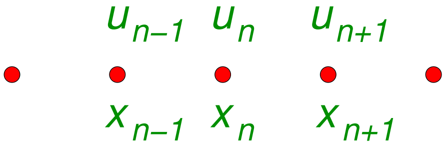

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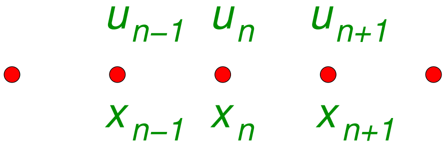

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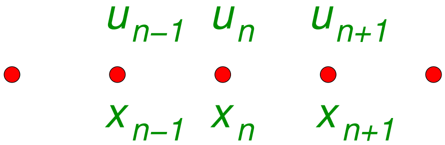

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- Evaluate F at the grid points x_n in terms of the u_k: $F(u_k, x_n, t)$.
- Solve the coupled system of ordinary differential equations,

$$\frac{du_n(t)}{dt} = F[u_k(t), x_n, t],$$

using standard numerical methods (e.g. Runge-Kutta).

Basic Numerical Methods

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- Most numerical groups use finite difference methods:
  - Uniformly spaced grids: $x_n - x_{n-1} = \Delta x =$ constant.
  - Use Taylor expansions,

$$u_{n-1} = u(x_n - \Delta x) = u(x_n) - \partial_x u(x_n)\Delta x + \partial_x^2 u(x_n)\Delta x^2/2 + \mathcal{O}(\Delta x^3),$$
$$u_{n+1} = u(x_n + \Delta x) = u(x_n) + \partial_x u(x_n)\Delta x + \partial_x^2 u(x_n)\Delta x^2/2 + \mathcal{O}(\Delta x^3),$$

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- Grid spacing decreases as the number of grid points N increases, $\Delta x \sim 1/N$. Errors in finite difference methods scale as N^{-p}.

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- Errors in spectral methods are dominated by the size of $\tilde{u}_N$.
- Estimate the errors (for Fourier series of *smooth* functions):

$$
\begin{aligned}
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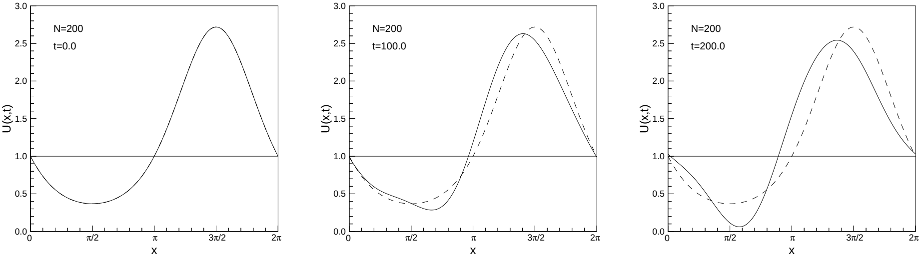
- Errors in spectral methods decrease faster than any power of N.

Comparing Different Numerical Methods

- Wave propagation with second-order finite difference method:



Figures from Hesthaven, Gottlieb, & Gottlieb (2007).

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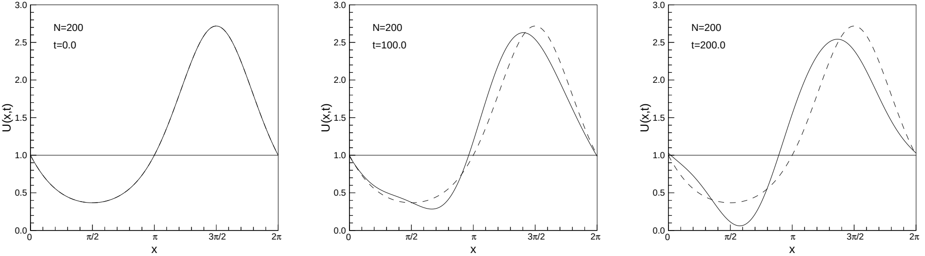

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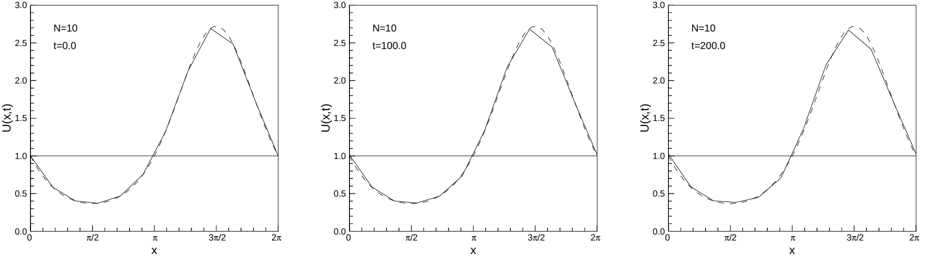

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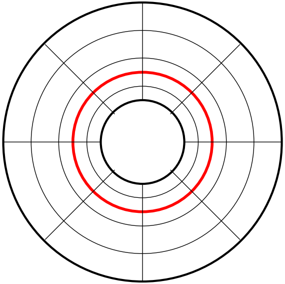
Moving Black Holes

- Black hole interior is not in causal contact with exterior. Interior is removed, introducing an excision boundary.



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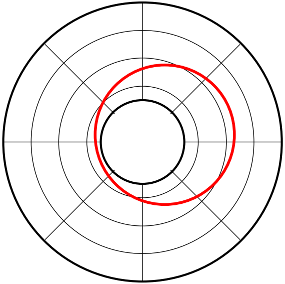

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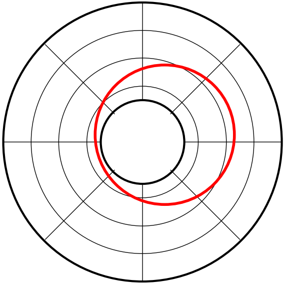

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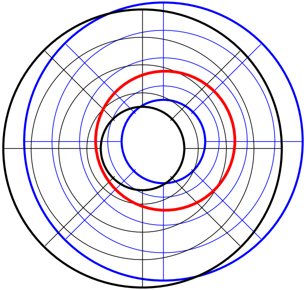

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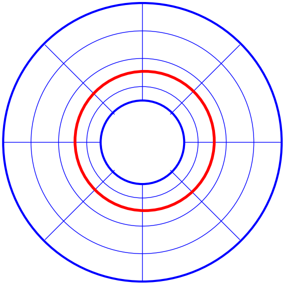

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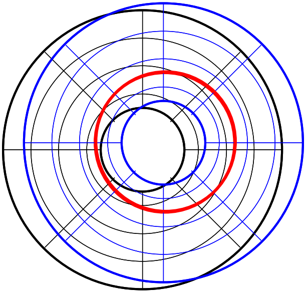

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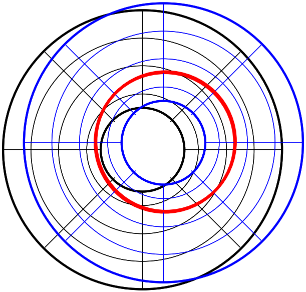

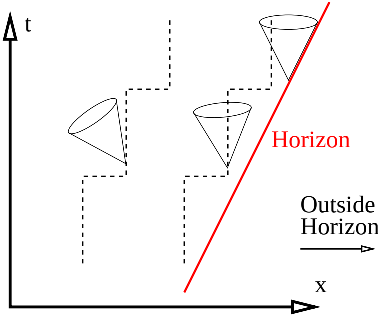

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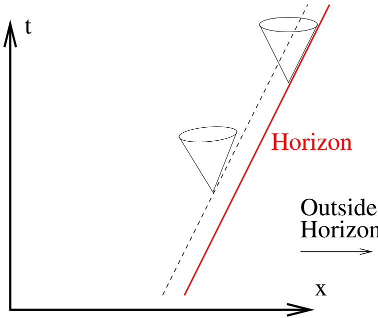
- Solution:
  Choose coordinates that smoothly track the motions of the centers of the black holes.



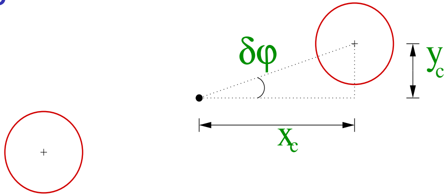
Horizon Tracking Coordinates

- Coordinates must be used that track the motions of the holes.
- A coordinate transformation from "inertial" coordinates, $(\bar{x}, \bar{y}, \bar{z})$, to "co-moving" coordinates (x, y, z), consisting of a rotation followed by an expansion,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{a(\bar{t})} \begin{pmatrix} \cos\varphi(\bar{t}) & -\sin\varphi(\bar{t}) & 0 \\ \sin\varphi(\bar{t}) & \cos\varphi(\bar{t}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix},$$

  is general enough to keep the holes fixed in co-moving coordinates for suitably chosen functions $a(\bar{t})$ and $\varphi(\bar{t})$.
- Since the motions of the holes are not known *a priori*, the functions $a(\bar{t})$ and $\varphi(\bar{t})$ must be chosen dynamically and adaptively as the system evolves.

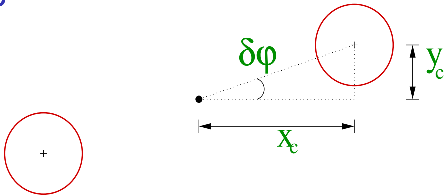
Horizon Tracking Coordinates II



- Measure the co-moving centers of the holes: $x_c(t)$ and $y_c(t)$, or equivalently

$$\begin{aligned} Q^x(t) &= \frac{x_c(t) - x_c(0)}{x_c(0)}, \\ Q^y(t) &= \frac{y_c(t)}{x_c(t)}. \end{aligned}$$

Horizon Tracking Coordinates II



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$$Q^y(t) = \frac{y_c(t)}{x_c(t)}.$$

- Choose the map parameters $a(t)$ and $\varphi(t)$ to keep $Q^x(t)$ and $Q^y(t)$ small.
- Changing the map parameters by the small amounts, δa and $\delta\varphi$, results in associated small changes in δQ^x and δQ^y:

$$\delta Q^x = -\delta a, \qquad \delta Q^y = -\delta\varphi.$$

Horizon Tracking Coordinates III

- Measure the quantities $Q^y(t)$, $dQ^y(t)/dt$, $d^2Q^y(t)/dt^2$, and set

$$\frac{d^3\varphi}{dt^3} = \lambda^3 Q^y + 3\lambda^2 \frac{dQ^y}{dt} + 3\lambda \frac{d^2Q^y}{dt^2} = -\frac{d^3Q^y}{dt^3}.$$

  The solutions to this “closed-loop” equation for Q^y have the form $Q^y(t) = (At^2 + Bt + C)e^{-\lambda t}$, so Q^y always decreases as $t \to \infty$.

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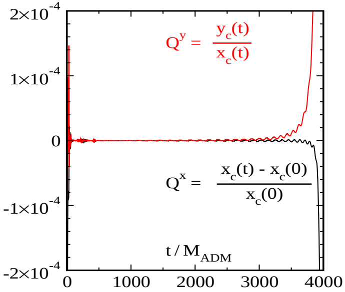
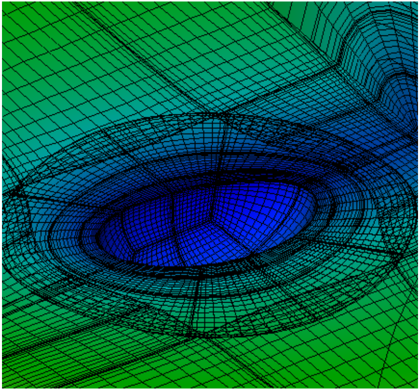
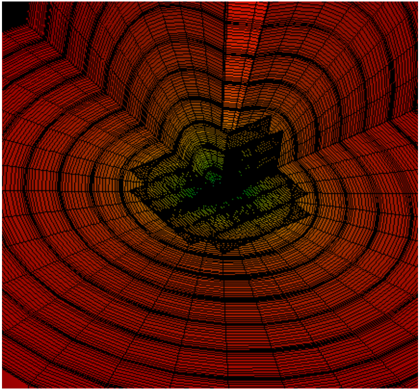
- **This works!** We are now able to evolve binary black holes using horizon tracking coordinates until just before merger.



Caltech/Cornell Spectral Einstein Code (SpEC):

- Multi-domain spectral method.





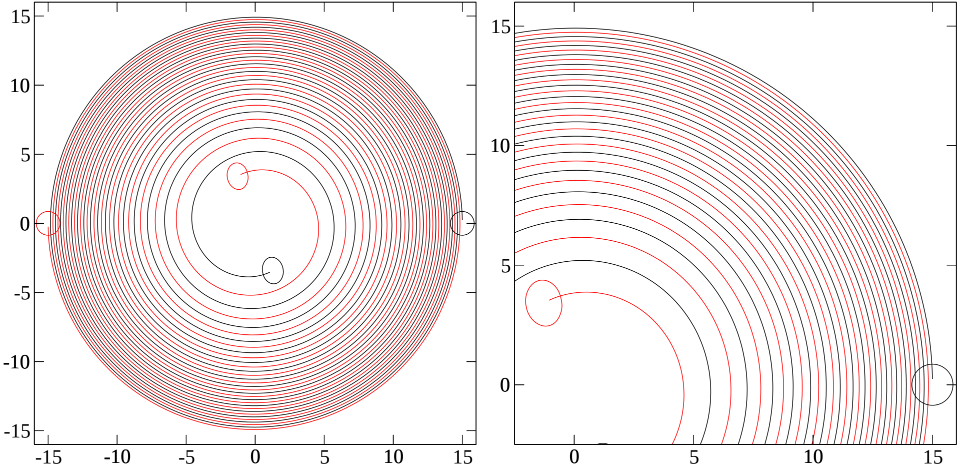
- State of the art elliptic solver for computing BBH initial data, etc.
- Constraint damped "generalized harmonic" Einstein equations:

$$\Box g_{ab} = F_{ab}(g, \partial g).$$

- Constraint-preserving, physical and gauge boundary conditions.

Evolving Binary Black Hole Spacetimes

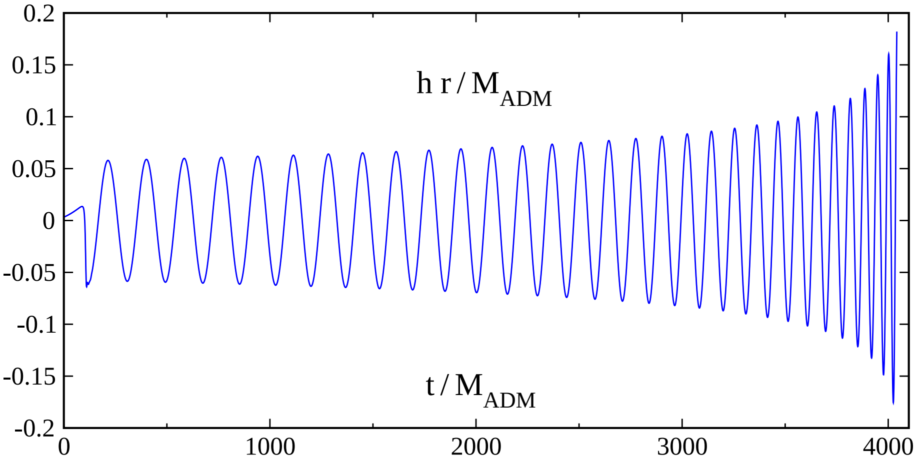
- We can now evolve binary black hole spacetimes with excellent accuracy and computational efficiency through many orbits.



Head-on Merger Movie

Gravitational Waveforms

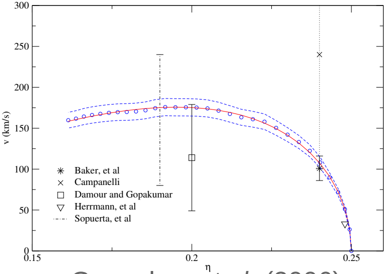
- High precision gravitational waveform for equal mass non-spinning BBH system.



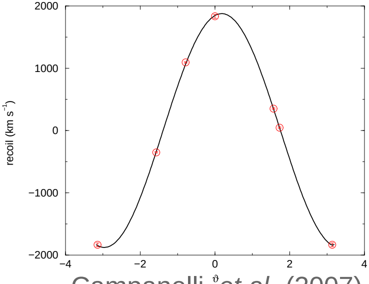
Ψ_4 Movie

Black Hole Recoil

- Asymmetric binaries (unequal masses and/or non-aligned spins) emit linear momentum into GW; final merged hole recoils with non-zero velocity.
- Maximum recoil velocity for non-spinning holes is 175 km/sec.
- Recoil velocities of 2000 km/sec have been measured in spinning black hole simulations (estimated maximum $\sim$ 4000 km/sec).
- Large recoils are probably rare (Schnittman & Buonanno 2007).



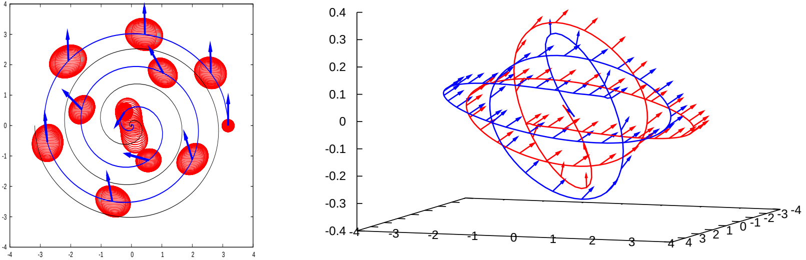
Gonzalez, *et al.* (2006)



Campanelli, *et al.* (2007)

Spin Dynamics in Binary Black Hole Systems

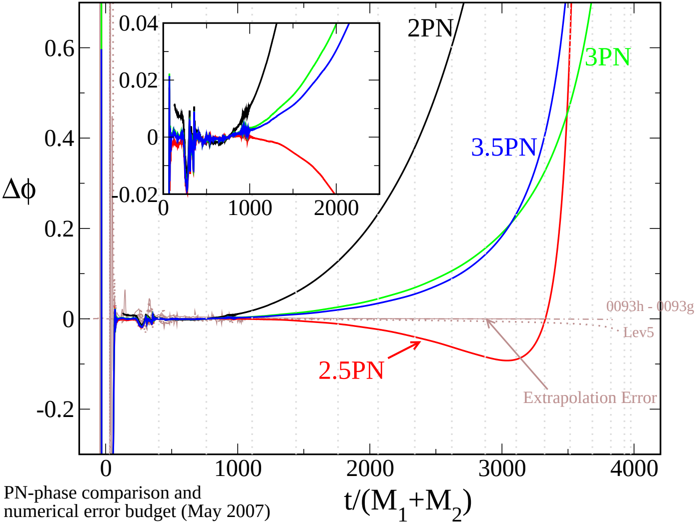
- Merger is delayed in BBHs with spins aligned with the orbital angular momentum; merger happens more quickly in binaries with anti-aligned spins.
- Complicated spin dynamics are observed in BBH mergers with non-aligned spins.



Campanelli, *et al.* (2007)

Post-Newtonian Waveform Calibration

- Preliminary comparisons of the numerical gravitational wave phase with predictions of various post-Newtonian orders.



PN-phase comparison and numerical error budget (May 2007)

Summary

- Advances in understanding the Einstein equations provide new formulations suitable for numerical evolutions: hyperbolic formulations with constraint damping and well posed initial-boundary value problems.
- High accuracy multi-orbit binary black hole simulations are now routine (but not yet cheap).
- Numerical waveforms suitable for LIGO data analysis are starting to be generated.
- Interesting non-linear dynamics of binary black hole mergers are beginning to be investigated.