Testing Post-Newtonian Waveforms with Numerical Relativity

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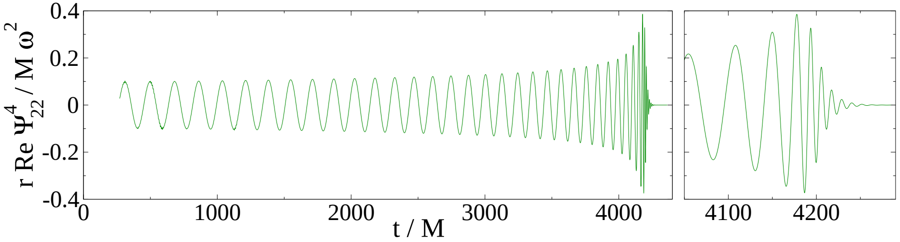
Collaborators:

Michael Boyle, Duncan Brown, Lawrence Kidder, Abdul Mroue, Harald Pfeiffer, Mark Scheel, Gregory Cook, and Saul Teukolsky.

Numerical BBH Gravitational Waveforms

- High precision numerical inspiral-merger-ringdown waveforms are now available for equal-mass non-spinning BBH systems.



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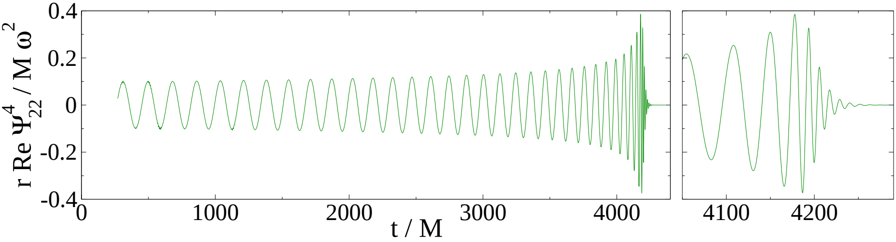

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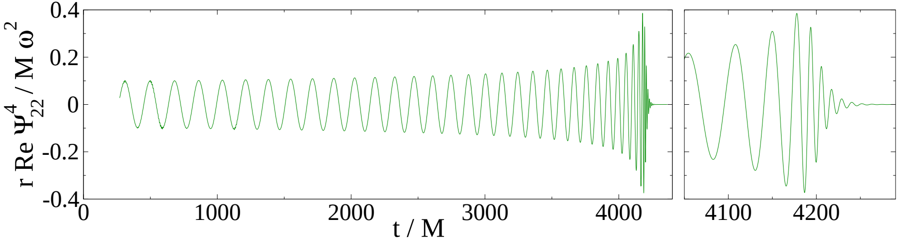

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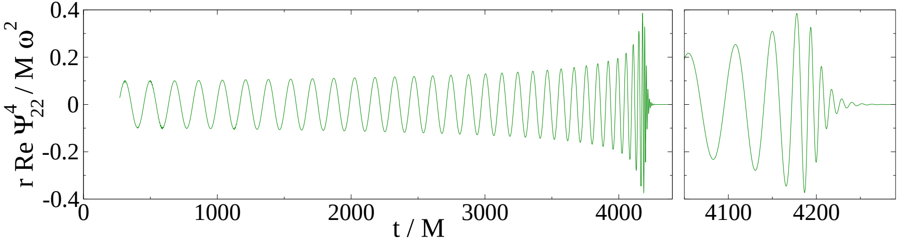
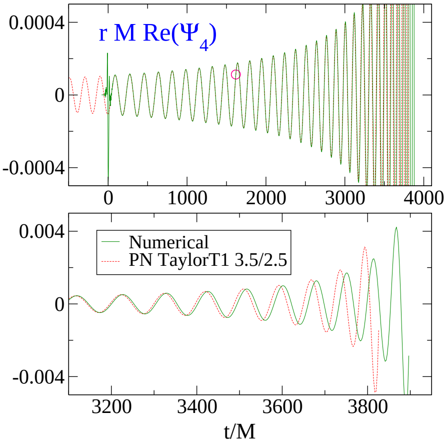


- Post-Newtonian (PN) waveforms are widely used to construct detection templates for LIGO data analysis.
- How accurate are PN waveforms?
- Test the PN waveforms by comparing them to the "exact" numerical relativity waveforms:
  Boyle, et al., *Phys. Rev. D*, **76**, 124038 (2007).

Comparison of Numerical with PN Waveforms

- PN waveforms are computed using the TaylorT1 method at order 3.5 in phase and 2.5 in amplitude.
- PN waveforms are matched to Numerical waveforms by adjusting time and phase offsets at the point where $\omega = 0.04/M$.



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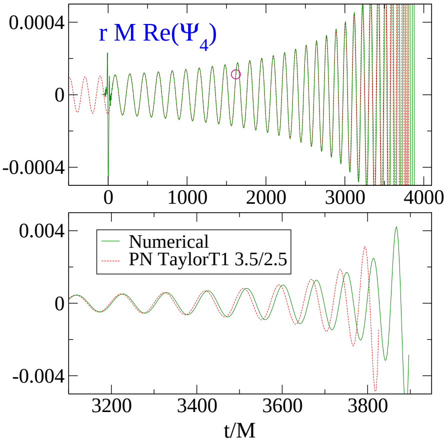

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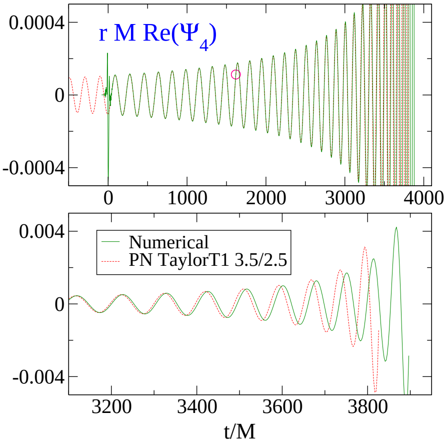
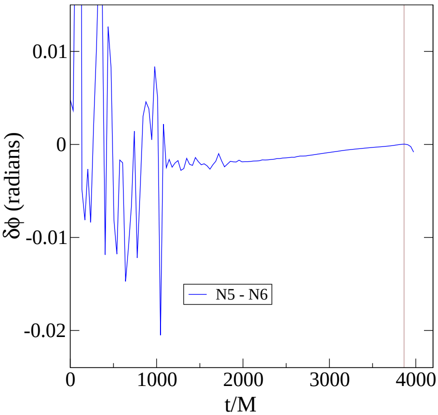
- Are the differences significant?
- How accurate are the Numerical waveforms?
- What is the TaylorT1 method, and does it matter?



Determining Numerical Waveform Accuracy



- Numerical convergence of gravitational waveform.

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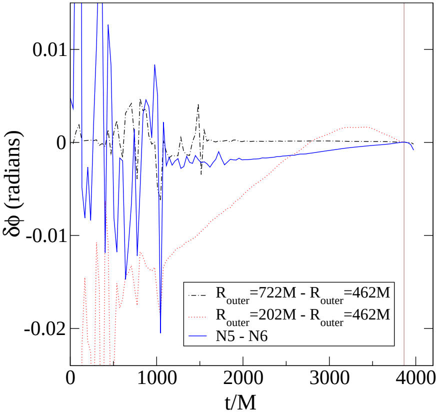

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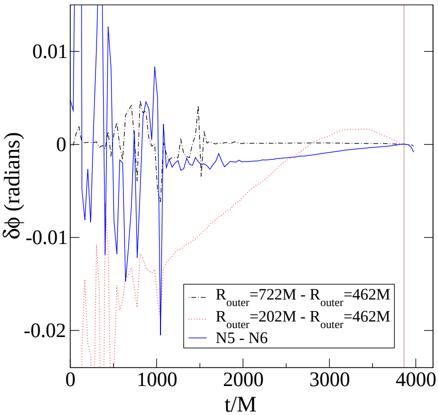

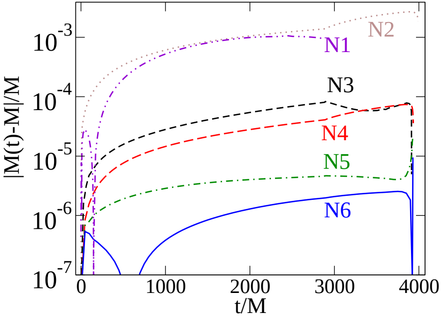

- Numerical convergence of gravitational waveform.
- Phase dependence on outer boundary location.
- Constancy of the black hole masses.

Summary of Numerical Waveform Phase Errors:

Effect	$\delta\phi$ (radians)
Numerical truncation error	0.003
Finite outer boundary	0.005
Drift of mass M	0.002
Extrapolation $r \to \infty$	0.005
Wave extraction at r_{areal}=const?	0.002
Coordinate time = proper time?	0.002
Lapse spherically symmetric?	0.01
root-mean-square sum	**0.01**

Summary of Numerical Waveform Phase Errors (Including Physical Parameter Errors):

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Extrapolation $r \to \infty$	0.005
Wave extraction at $r_{\rm areal}$=const?	0.002
Coordinate time = proper time?	0.002
Lapse spherically symmetric?	0.01
residual orbital eccentricity	0.02
residual black hole spin	0.03
root-mean-square sum	**0.04**

Post-Newtonian Gravitational Waveforms

TaylorT1

1. Rewrite energy-balance equation

$$\frac{dE_{\mathrm{binary}}}{dt} = \frac{dE_{\mathrm{binary}}}{d\Omega}\frac{d\Omega}{dt} = -\frac{dE_{\mathrm{GW}}}{dt} \quad \Rightarrow \quad \frac{d\Omega}{dt} = -\frac{dE_{\mathrm{GW}}/dt}{dE_{\mathrm{binary}}/d\Omega}$$

2. Substitute Taylor series from PN expansion on right-hand side

$$\frac{d\Omega}{dt} = -\frac{\Omega^{10/3}\left(A_0 + \ldots + A_n\Omega^{n/3}\right)}{\Omega^{-1/3}\left(B_0 + \ldots + B_n\Omega^{n/3}\right)}$$

3. Numerically integrate once to find Ω
4. Numerically integrate once more to find Φ

Post-Newtonian Gravitational Waveforms

TaylorT4

1. Rewrite energy-balance equation

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3. Re-expand right-hand side as a Taylor series, and truncate

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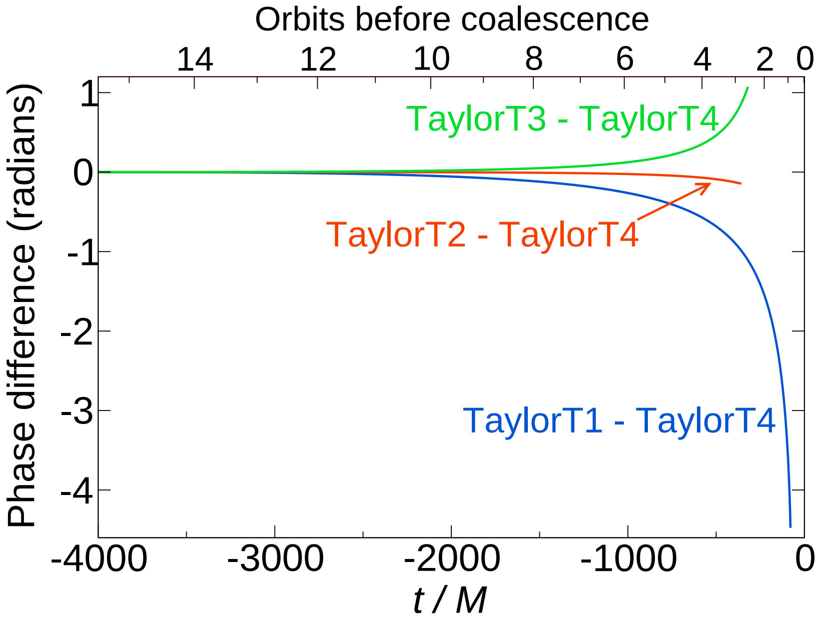
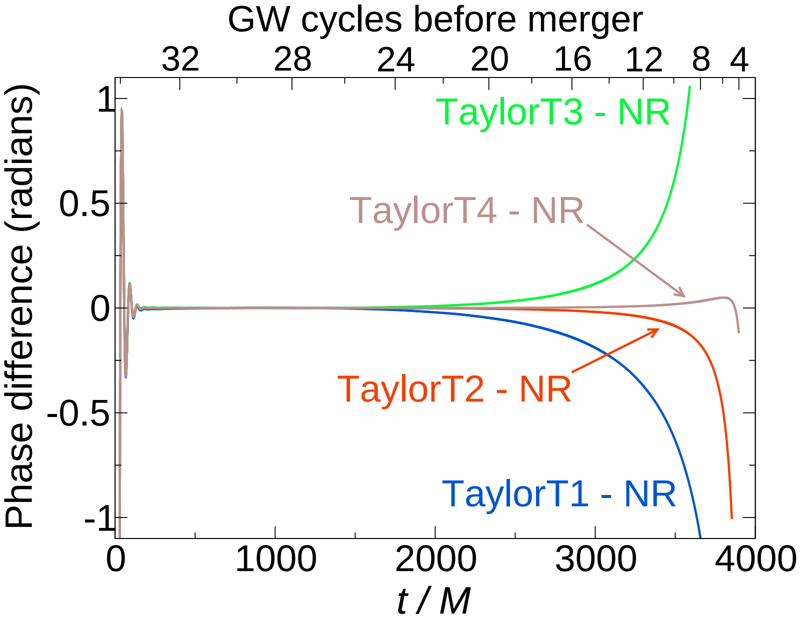
TaylorT2, ***TaylorT3***, ...

Comparing Various PN Methods



Comparing Various PN Methods with NR Waveform



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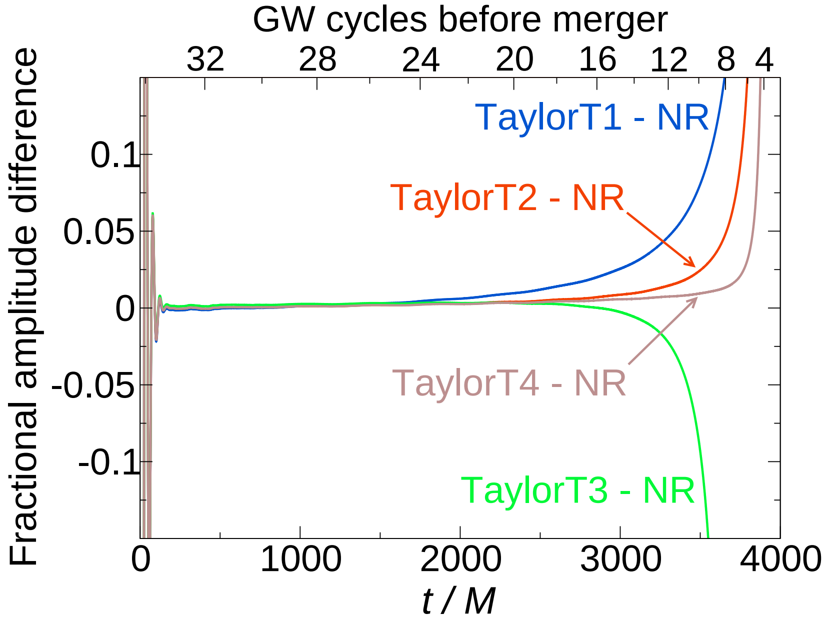

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The End!

Thanks to Don Marolf and the UCSB organizers!