Introduction to Binary Black Hole Evolutions

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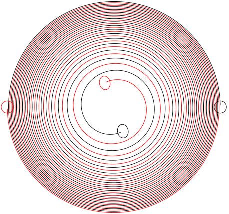
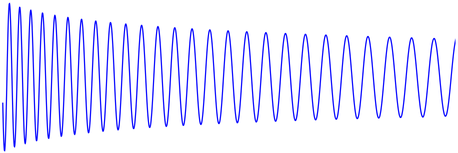
23rd Pacific Coast Gravity Meeting
17 March 2007





Generalized Harmonic Einstein Equations

- Specify the spacetime coordinates x^a using a gauge source function $H^a(x,\psi)$:

$$H^a(x,\psi) = \nabla^c\nabla_c x^a = \psi^{bc}\partial_b\partial_c x^a + ..,$$

 where ψ_{ab} is the spacetime metric.

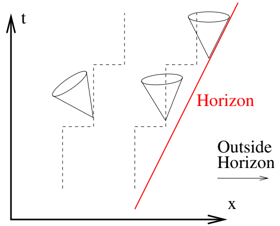
- The Einstein equations become manifestly hyperbolic:

$$\psi^{cd}\partial_c\partial_d\psi_{ab} \;=\; \nabla_a H_b + \nabla_b H_a + F_{ab}(\psi,\partial\psi).$$

- Adding certain multiples of the constraints to these equations gives them excellent constraint damping properties.

Moving Black Holes

- Problem: Causality issues when black holes move through a computational domain:



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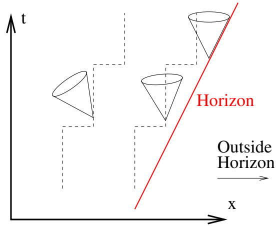
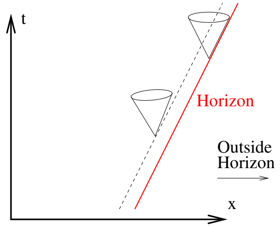


- Solution: Choose coordinates that smoothly track the location of the black hole.



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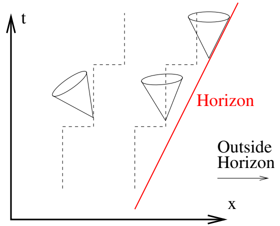

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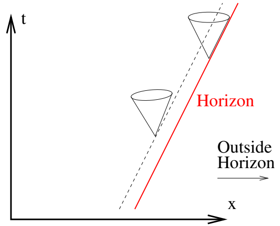
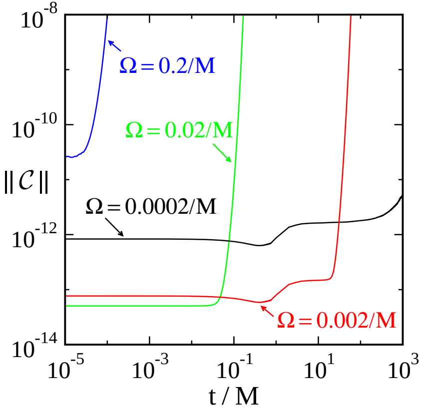


- Use co-rotating coordinates for widely separated binaries.

Evolving Black Holes in Rotating Frames

- Evolutions of Schwarzschild in rotating coordinates are unstable.



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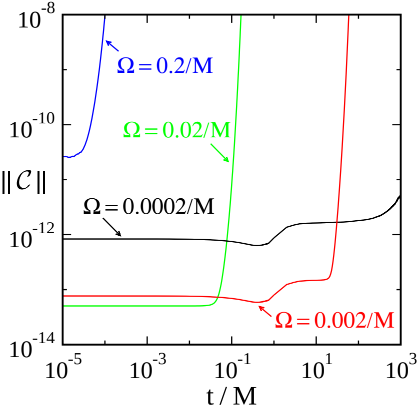

- Problem caused by asymptotic behavior of metric in rotating coordinates: $\psi_{tt} \sim \rho^2\Omega^2$, $\psi_{ti} \sim \rho\Omega$, $\psi_{ij} \sim 1$.

Dual-Coordinate-Frame Evolution Method

- Coordinates serve two different purposes:
  - The coordinate basis defines the components of tensor fields:

$$\psi = \psi_{ab}\, dx^a \otimes dx^b.$$

  - Field components are determined as functions of the coordinates:

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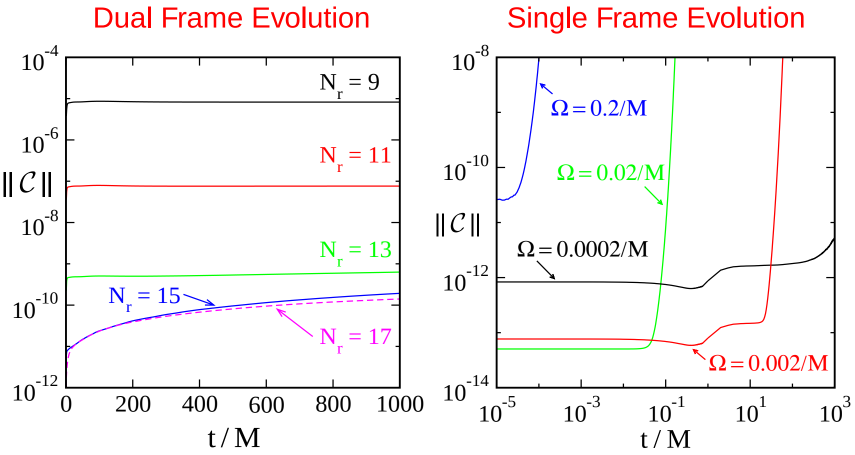
- Use different coordinates for different purposes:
  - Use asymptotically inertial coordinates to define field componets.
  - Use co-moving coordinates to evaluate these functions by solving Einstein's equation.

Testing Dual-Frame Evolutions

- Single-frame evolutions of Schwarzschild in rotating coordinates are unstable, while dual-frame evolutions are stable:

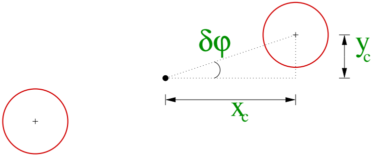


Horizon Tracking Coordinates

- Choose co-moving coordinates that track the black holes:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e^{a(\bar{t})} \begin{pmatrix} \cos\varphi(\bar{t}) & -\sin\varphi(\bar{t}) & 0 \\ \sin\varphi(\bar{t}) & \cos\varphi(\bar{t}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}.$$

- Measure the comoving positions of the holes:



$$Q^x(t) = \frac{x_c(t) - x_c(0)}{x_c(0)},$$

$$Q^y(t) = \frac{y_c(t)}{x_c(t)}.$$

- Choose the map parameters $a(t)$ and $\varphi(t)$ to keep $Q^x(t)$ and $Q^y(t)$ small.

Horizon Tracking Coordinates II

- In the time interval $t_i < t < t_{i+1}$ choose the map parameters:

$$\begin{aligned}\varphi(t) &= \varphi_i + (t-t_i)\frac{d\varphi_i}{dt} + \frac{(t-t_i)^2}{2}\frac{d^2\varphi_i}{dt^2} \\ &\quad + \frac{(t-t_i)^3}{2}\left(\lambda\frac{d^2Q^y_i}{dt^2} + \lambda^2\frac{dQ^y_i}{dt} + \lambda^3\frac{Q^y_i}{3}\right).\end{aligned}$$

- This choice drives $Q^y(t)$ exponentially toward zero on the timescale $1/\lambda$.

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- This choice drives $Q^y(t)$ exponentially toward zero on the timescale $1/\lambda$.
- **This works!** We are now able to evolve binary black holes using horizon tracking coordinates until just before merger.